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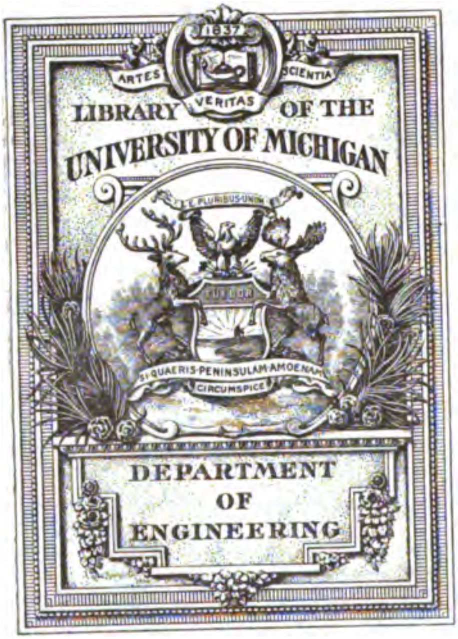

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PURE MECHANISM

A TEXTBOOK OF
PURE MECHANISM

BY

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PREFACE

The discussion in this book does not cover a large number of mechanical appliances, not even a large number of those to be found in common use. An attempt has been made to select representative examples which best illustrate the geometry of machinery and to state them as briefly as possible without a sacrifice of clearness.

The classification is based on the method of transmitting motion. This is radically different from the classification used in several well-known text-books which is according to the method of making contact, *i.e.*, point, line or surface. The former method seems to lead to a somewhat less complex treatment of the subject.

In Chapter III, certain fundamental propositions are stated as true of all contact motions and examples in the later parts of the book are constantly referred back to them.

The method of treating cams and spiral gears is original so far as the author knows. The material therefor, as well as for the other parts of the book, was culled from existing writings and represents comparatively little experimental work. The current magazines, trade catalogs and the works of other authors have been drawn upon for ideas and illustrations and credit is hereby given. Special mention is due the works of Rankine, MacCord, Unwin, Reuleaux, Robinson, Stahl & Woods, Barr, Merrill & Schwamb, Brown & Sharpe, Anthony, Grant, Halsey and Logue, the class-room lectures of C. H. Benjamin and the columns of *Machinery* and the *American Machinist.*

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PURE MECHANISM

CHAPTER I

DEFINITIONS

1. Mechanics of Machines or Mechanism is that branch of Applied Mechanics which treats of the design, construction and operation of machinery.

The subject has two divisions: *Pure Mechanism,* sometimes called Kinematics of Machinery, and *Constructive Mechanism* or Machine Design.

Pure Mechanism deals principally with the form and motion of machines; machine design involves questions of the weight and strength of parts and of the forces in operation.

The word "mechanism" is used in two senses, first as a general term indicating a branch of the science of Mechanics. It is also used to indicate a body of special shape, as in "a piece of mechanism," or a group of such bodies so arranged as to transmit motion.

In this last sense there is little difference between the definition of a mechanism and a machine, but the word machine implies completeness, something that will do work; this element may be wholly lacking in a piece of mechanism.

2. A Machine is a combination of resistant bodies, fixed or movable, arranged to transmit and modify motion and energy and to do work. Two things are pre-supposed, a source of power and work to be done. The machine is interposed between these two, and its working members may be classed under three heads:

(a) Parts receiving the energy,
(b) Parts transmitting and modifying the energy
(c) Parts performing the required work.

To this might be added two more headings:

(d) Regulators,
(e) Supports or frames.

The steam engine, Fig. 1, serves to illustrate the above classification. The piston P receives motion from the expansive force of the steam. The cross head H, the connecting rod R, and crank C transmit the motion to the shaft S and fly-wheel D. These parts in turn do the work, usually on some secondary machine. The bed A supports the parts in their proper relative positions and the governor B and the valve V regulate the steam supply and control the speed.

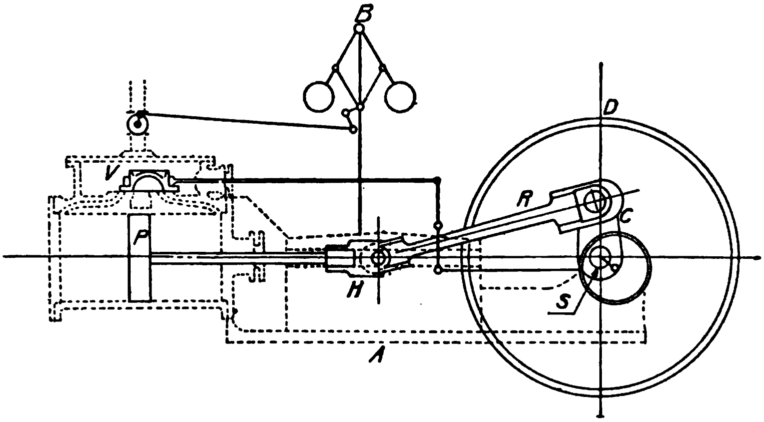


FIG. 1.

3. Motion, in the mechanics of machinery, is understood to mean the movement of one part of a mechanism relative to the other parts, the change of position being definite in direction and velocity.

4. Path.—If a point changes its position, it traces a line called a path. A path may be of any form whatever, in a plane or in space. There are, however, only a few forms of paths that are made use of in machine design; straight lines and closed loops being the most common. If the path of a moving point and the direction and velocity along the path are given, the motion is completely determined.

5. Cycle, Period, Phase.—When a point, which is moving in a given path, makes a complete circuit and returns to the original position, such a circuit is called a cycle. The time required for one such complete cycle is the *period* of the moving point, and by the *phase* is meant the position of the point on the path, or the angle turned through at a particular instant. In Fig. 2, if P is moving in the circumference about O, one complete revolution is a cycle, the time required for one revolution is the period, and the phase is given by the angle θ.

6. Forms of Motion.—Motions available for machine design may be classified with respect to the form of the path under three heads:

(a) Plane motion $\begin{cases}\text{Rotation.}\\ \text{Translation.}\end{cases}$
(b) Helical motion.
(c) Spherical motion.

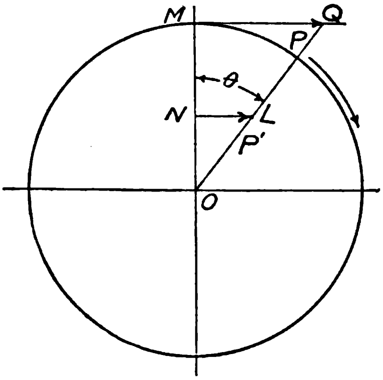


FIG. 2

7. Plane Motion.—Let a section of a rigid moving body be made by a cutting plane, the plane is extended if necessary in the direction of motion; if the section of the moving body so cut always remains in this cutting plane or its extension, then the body has *plane motion.* Plane motion may be either rotation or translation. If it is rotation, the body moves around a fixed center located either within or without the body itself. A plane motion of translation may be either rectilinear, in which case its path is a straight line, or curvilinear, where the body moves in such a way as to remain always parallel to its initial position. The wheels of a locomotive furnish an example of plane motion of rotation; the frame of the engine when moving on a straight track has rectilinear translation and the side rods are an illustration of curvilinear translation.

8. Helical Motion.—If a body has rotation around an axis combined with rectilinear motion along the axis, the resultant motion is *helical.* A point moving along a screw thread would have helical motion.

9. Spherical Motion.—If a body revolves in two directions at the same time about a fixed point, located outside of the body itself, its motion is *spherical.* The motion of the governor balls, *B,* Fig. 1, belongs under this classification.

10. Relations between Plain, Spherical and Helical Motion.—Helical and spherical motions are combinations of two simultaneous plane motions at right angles to each other. Helical motion is rectilinear translation along an axis combined with a plain rotation about the axis.

The governor balls, already mentioned, as an illustration of spherical motion, have plane rotation about a horizontal axis and plane rotation about a vertical axis.

11. Revolution and Rotation.—When a body has plane motion around an axis which is located some distance from the body the latter is said to revolve about the axis.

If all the points in a body have plane motion around an axis which passes through the body itself, the body is said to rotate about that axis. For example, the earth revolves around the sun and rotates on its axis.

12. Direction.—Motion in a straight line is either upward or downward, or to the right or left. Such motion must be reversed periodically or the moving body would continue on to infinity. When motion takes place in a closed loop or path, it may be constant in one direction or it may be reversible. A reversal in direction generally assumes a reversal in the applied force but this is not always the case as may be seen by Fig. 3, a mechanism known as a mangle wheel. The wheel, being driven by the pinion, will turn in the direction indicated by the arrow until the last pin *F* reaches the pinion; the latter will then drop down in its supporting slot and the direction of the wheel be automatically reversed while that of the pinion remains the same.

13. Velocity, is the distance passed over by a moving body in any unit of time. It is commonly reckoned in inches per second, feet per minute, or miles per hour. By formula, velocity $= \frac{\text{distance}}{\text{time}}$. Velocity is said to be variable or uniform according as it changes from one instant to another or remains constant. When velocity is varying, the average or mean velocity is the whole distance passed over, divided by the time of passage; this may vary considerably from the velocity at any particular instant. Thus, if a train moves between two stations located 40 miles apart, in an hour, its mean velocity is 40 miles an hour, while its actual velocity, at a particular place on the road, may be much greater.

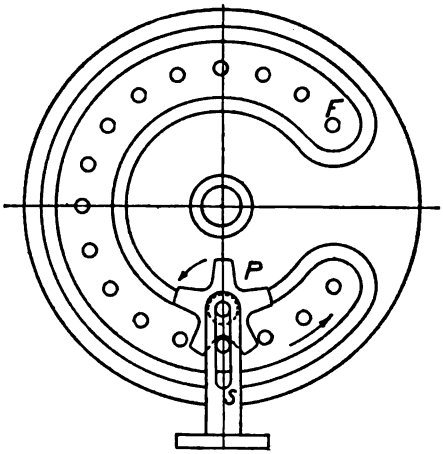


FIG. 3.

14. Acceleration is the rate of change of velocity. Thus, if a body is moving at a given instant with a velocity V_o, and at the end of t seconds with a velocity V_t, the change of velocity is $V_t - V_o$, and the rate of change, or the acceleration, is $\frac{V_t - V_o}{t}$.

If a body has a velocity of X feet per second, its acceleration will be expressed by X feet per second per second.

Acceleration may be either positive or negative, that is, the speed of a body may be increasing or decreasing.

15. Linear and Angular Velocity.—The linear velocity of a moving point is its velocity in linear dimensions along any path in space. The motion of the piston in Fig. 1 is an example of linear velocity.

When motion takes place around a center or an axis, it is convenient to express its rate in terms of angular velocity. The unit of angular velocity is the radian, the radian being a circular arc equal to the radius of the circle. A circumference being equal to $2\pi r$, contains 2π, or 6.283 radians and each radian will subtend an angle of 57.3° at the center.

It follows from the above definition that the angular velocity of a point is equal to its linear velocity divided by the radius of rotation. Thus, suppose a point makes N rotations per minute about an axis located R feet away. Then its linear velocity will be $2\pi\ RN$ feet per minute and its angular velocity will be:

$$\frac{2\ \pi\ RN}{R} = 2\ \pi\ N \text{ radians per minute.}$$

In any rotating body, a fly-wheel for example, if the angular velocity is constant, the linear velocity of any point in the body will vary directly as its distance from the center of rotation. In Fig. 2, if θ represents the angular velocity of a body rotating about O, at a given instant and MQ the corresponding linear velocity of a point P, then NL would represent the linear velocity of this point if it were located at some other position as P'. But from similar triangles, $MQ : NL :: OM : ON$. It follows then, that there may be any number of linear velocities in a rotating body, but the angular velocity at any instant can have but one value.

16. Continuous, Intermittent and Reciprocating Motion.—If motion continues indefinitely in a fixed path without a reversal in direction, such motion is said to be *continuous*. The fly-wheel in Fig. 1 has continuous motion. When motion is interrupted by periods of rest the motion is *intermittent*. The motion of the governor balls in a vertical plane, in the same figure, is intermittent. If motion takes place over a fixed path, in a given direction, for a given length of time, and then reverses its direction, moving over the same path in the opposite direction it is said to be *reciprocating motion*. The cross head and piston of Fig. 1 have reciprocating motion.

17. Driver and Follower.—When two bodies are moving in contact so as to transmit motion from one to the other, the body that imparts the motion is called a *driver* and the body which receives motion from the driver is the *follower.*

18. Free and Constrained Motion.—All motion is either *free* or *constrained.* A body which has no rigid connection with other bodies has free motion. The planets are examples of free motion, since every disturbing force acting upon them alters their paths to a certain extent. A body which has a rigid connection with other bodies, thereby restricting its motion to a definite path, is constrained. All motions available for machine design are constrained motions. Constrainment is attained by *pairing.* A machine is made up of a definite number of pairs of elements, the members of each pair being so related that the fixed one prevents every motion of the movable one except the motion desired. One member of a pair is the envelope of the other, the shape of the envelope depending upon the nature of the motion. When all motion of the constrained element except the one desired, is prevented, the pair is *closed.* Thus in Fig. 1, the shaft S is constrained by its block so that it can have no motion except that of rotation on its axis. Kinematically, it makes no difference whether the movable element of a pair or the fixed one is the envelope. Thus the shaft may move and the envelope remain fixed or the reverse may be the case.

19. Open or Force Closed Pair.—When one element of a pair is not completely constrained but only enough to resist forces acting in a certain direction, the pair is an *open* one. The track rail and wheel of a locomotive form an open pair, the constraining power of the envelope is in this case assisted by the force of gravity, hence pairs such as this are sometimes called *force closed* pairs.

20. The Contact, existing between any pair of elements may have any one of three forms: (1) point, (2) line, (3) surface. In point contact the surfaces touch each other only in a single point as when two spheres are in contact in ball bearings.

If two cylinders, having their axes parallel, be placed in contact they will touch each other along a straight line. Two elements in line or point contact are sometimes termed a *higher pair.*

When two elements are in contact over a definite area they are in surface contact and in some systems of classification form what is known as the *lower pair*.

21. Object of Kinematical Analysis.—This is to investigate the form and contact of bodies and the form, direction, velocity and acceleration of their motion.

22. Classification of the Subject.—Motion may be transmitted from a driver to a follower by *direct contact*, the bodies, touching each other on a point, line or surface or the motion may be transmitted by *indirect contact*, a third piece or connector being used to join them together.

Each of these methods may be subdivided under two heads, thus: The action between two pieces in direct contact may be either *rolling* or *sliding*.

If a third piece is used between driver and follower, it may have the form of a *rigid link* like the side rod of a locomotive, or a *flexible band* like a leather belt.

Two systems have been used to classify the elements of pure mechanism. In one system the classification is based on the method of contact and mechanisms are grouped according as they contain one, or more than one combination of either higher or lower pairs.

The other system and the one which is adopted in this book, classifies mechanisms according to the method of transmitting motion.

Thus:

Modes of transmitting motion.	By intermediate connectors.	Links.
		Wrapping connectors.
	By direct contact.	Rolling.
		Sliding.
		Mixed.

PROBLEMS

1. B and C are two stations located 40 and 70 miles respectively from station A. If a train runs from A to B in 80 minutes and from B to C in 60 minutes what is the mean velocity between A and C.

2. If the above train starts from rest at station A and attains a velocity of 60 miles an hour at the end of the fifth minute, what is the average acceleration?

3. If the brakes are applied while running at a maximum velocity of 90 miles per hour, two minutes before reaching station C, what is the negative acceleration?

4. A fly-wheel 10 ft. in diameter has a rim velocity of 4000 ft. per minute. What is its angular velocity?

5. What is the linear velocity of a point 4 ft. from the center of the wheel

6. Find the linear and angular velocity of a point on the rim of a 5 ft. driving wheel of a locomotive when the engine is running 48 miles per hour.

CHAPTER II

GRAPHICAL ANALYSIS OF MOTION

23. Motion Diagrams.—The three properties of motion, direction, velocity and acceleration, may be represented by lines, the direction of the motion being shown by the direction of the line and velocity and acceleration by the length of the line.

The scale used may be in feet per minute or inches per second or any other combination of time and distance that is convenient. The scale for angular motion is in degrees per unit of time. In rectilinear motion, the speed may be scaled off directly along the path. If the motion is curvilinear it can be shown by the tangent to the curve along which a body is moving at any instant, the length of the tangent representing the speed at that instant.

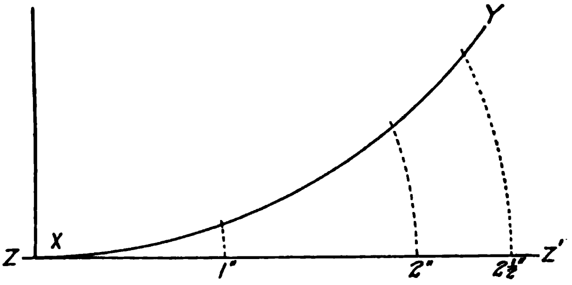


Fig. 4.

In Fig. 4 let a point be moving along a straight path, ZZ', at the rate of 50 ft. per minute, and suppose a scale of 20 ft. to the inch is chosen, then 2½ in. laid off on ZZ' represents this motion.

If the point is moving along the curve XY at the same speed as before, its velocity at the instant of passing Z can be shown by scaling off 2½ in. on the tangent ZZ'.

The figure used for the graphical representation of motion is called a motion diagram, it may be an irregular curve or a polygon of any number of sides. The motion diagram has two uses, one being to determine unknown properties, the other to show the form of some unfamiliar motion by representing it in relation to one that is more familiar.

Graphical solutions are only approximate but by choosing a coarse scale they are sufficiently accurate for practical purposes. Their advantage lies in the appeal to the sense of proportion, through the eye, and errors are thus easily detected. It is well, in analyzing motions to calculate them analytically and then check by the graphical methods.

24. Resolution of Motions.—In Figs. 5, 6 and 7, if motion takes place in the direction *AC*, the extent and velocity of the motion in the directions *AX* and *AY* may be required. *AB* and *AD* are called the components of *AC* and the process of finding them is called the resolution of motions. Where a motion is given to find its components, three cases present themselves:

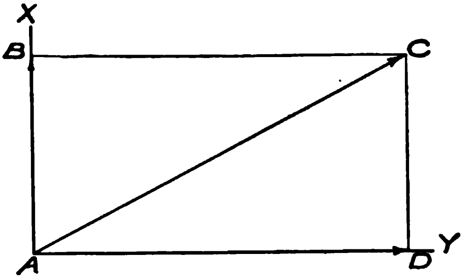


FIG. 5.

(a) Direction of two components known,
(b) Magnitude and direction of one component known,
(c) Magnitude of two components known.

Case (a), Fig. 5. Let the motion be represented by the line *AC*. Draw the indefinite lines *AX* and *AY* in the known direction of the components. Through *C*, draw a line parallel to *AX*, cutting *AY* at *D*. Also through *C* draw a line parallel to *AD*, cutting *AX* in *B*. *AB* and *AD* are the required components.

Case (b), Fig. 6. The known motion is given by the line *AC*.

Draw the indefinite line *AY* in the direction of the known component and lay off on it the known distance *AD*. Connect *CD*. Draw *AX* parallel to *CD*, and through *C* draw a line parallel to *AD*, cutting *AX* in *B*. Then *AB* is the other component.

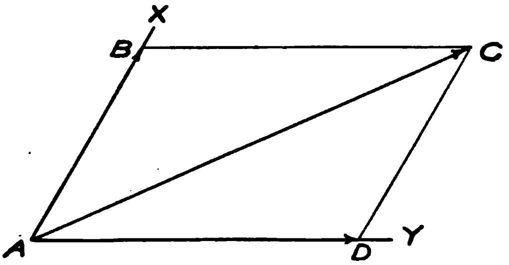


Fig. 6.

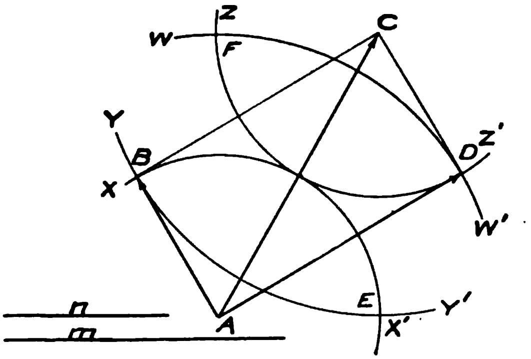


Fig. 7.

Case (c), Fig. 7. Two solutions are possible for this case. Let the lines *m* and *n* represent the length of the two components of *AC*.

With *A* as a center and radius equal to *m*, draw an arc *WW'*; and with *A* as a center and *n* as a radius, draw another arc *XX'*. Then with *C* as a center and radii equal to *m* and *n*, draw the arcs *YY'* and *ZZ'* intersecting the first pair of arcs in *B* and *D*. Draw the parallelogram *ABCD*. Then *AB* and *AD* will give the position of the components sought.

The second construction is similar to this except that the intersections E and F, on opposite sides of AC, are used to construct the figure.

Figs. 5, 6 and 7 are called parallelograms of motion.

To find the components of a motion, it is generally unnecessary to construct the whole parallelogram. Since in the figures, CD equals BA, it is only necessary, after laying out AC and AD, to connect CD. The triangle ADC equals the triangle $ABC = \frac{1}{2}\ ABCD$.

This is called the triangle of motions, its use saves the time and labor of drawing the whole figure.

25. Composition of Motions.—When the extent of a motion in the direction of two axes is known the actual motion may be found by a reversal of the method just described. The actual motion is called the resultant of the motions in the separate axes and the process of finding the resultant of a number of separate motions is known as the composition of motions.

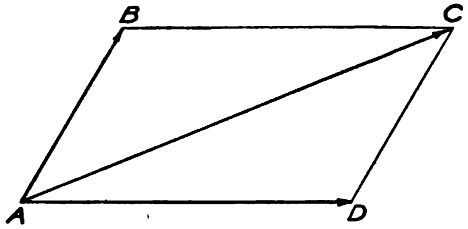


FIG. 8.

In Fig. 8, let AB and AD be two motions, known in amount and direction, laid off to any convenient scale. Through B draw BC parallel to AD, and through D draw DC parallel to AB, thus forming the parallelogram $ABCD$. The diagonal AC is the resultant of the given motions.

If more than two motions in one plane are given, the resultant can be found by forming a parallelogram with any two of the motions as sides, its diagonal can then be combined with one of the remaining motions and so on until all of the given motions have been involved.

In Fig. 9, let AB, AC, AD, and AE be given to find their resultant. First combining AB and AC, we get AX. AX is then combined with AD and the resultant is AY. Then AY can be combined with AE, giving the parallelogram $AYZE$. The diagonal AZ is the final resultant of the system.

From this discussion, it follows that a motion may have any number of components but several components can have only one final resultant.

In whatever order the components are combined, the final resultant will be the same; thus, AB may be combined with AC, AD, or AE, and the resultant so found again combined with one of those remaining. The final resultant will still be AZ

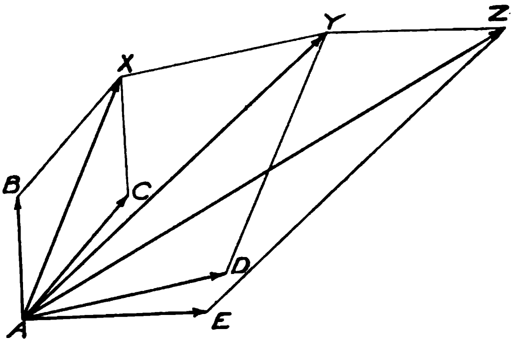


Fig. 9.

It follows from the examples given, that, in combining two components into one resultant, or resolving a motion into two components, six elements are involved, namely; the directions and velocities of the three motions. If four of these elements are given, the other two may be found.

26. The Instant Center.—When a point is moving in a plane its motion at a given instant is equivalent to rotation around some other point in the plane. This momentary center of rotation is called an *Instant Center* and its location is anywhere on a line drawn through the moving point and perpendicular to its instantaneous direction.

In Fig. 10 if a point is moving in the direction ZT, its motion, at the instant of passing X, is equivalent to a rotation about any center as O, on OX, the perpendicular to ZT.

If the point is moving in the curve XY, its direction, at the instant of passing X, is along ZXT, the tangent to XY at the point X. And again, its instant center can be located on OX. Hence, *to find the instant center of a moving point:* draw a tangent to the path along which the point is moving at the instant taken, erect a perpendicular to the tangent, at the point, and locate the instant center anywhere on the perpendicular so drawn.

If XY is an arc of a circle, O will be a fixed center, that is, the center of the circle of which XY is an arc. If XY is not a circular arc, the position of O changes whenever the radius of curvature of XY changes. The path traced by this moving instant center is called a *centrode.* Stating it another way, a centrode is the locus of the instant centers of a moving point.

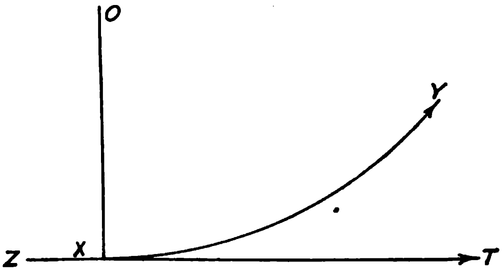


FIG. 10.

27. Instant Axis.—If a line be passed through the instant center of a moving point, perpendicular to its plane of motion, it will be an *instant axis* of the point.

In Fig. 11, a point moving in the path XY has the direction of XT perpendicular to the plane $ABCD$, at the instant of passing X. Its instant center can be located on any line drawn through X and lying in the plane $ABCD$, as at O, O' or O'' Its instant axis will be a line drawn through any one of these points and lying in the plane $ABCD$.

In the general case of a point moving anywhere in space it is necessary to refer the motion to one or more instant axes instead of a center.

28. Simultaneous Axes.—Two or more points moving in a plane may have a common instant center. Thus in Fig. 12 let the points P and P' be moving in the directions indicated by V and V'. Draw ZQ, perpendicular to PV and YQ perpendicular to $P'V'$. Then the instant center of P may be anywhere on ZQ and the instant center of P' may be anywhere on YQ. The common instant center therefore lies at Q the intersection of ZQ and YQ.

If the motion of a third point P'' in the same plane is required, it may be determined by drawing a line from the point to the instant center and erecting a perpendicular to the line at the point.

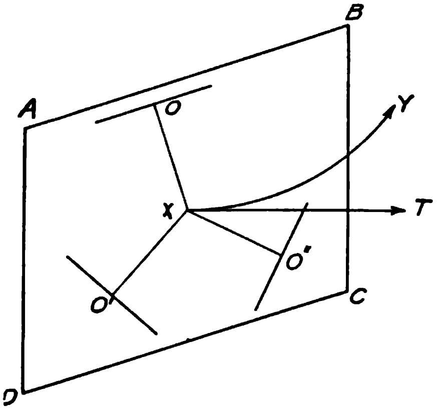


Fig. 11.

When the direction of the moving points is parallel as in Fig. 13 (*a*) the instant center is an infinite distance away and therefore indeterminate. If however the motions are parallel but opposite in direction as in Fig. 13 (*b*) the instant center, Q will lie between them.

If the points are not all in the same plane their motion must be referred to a common axis through Q. The location of such an axis is shown in Fig. 14. The points P and P' are anywhere in space a finite distance apart and moving in any directions as PV and $P'V'$. Pass planes through P and P' perpendicular to the directions of motion. Then the instant axis of P may be anywhere in the plane $ABCD$; also the instant axis of P' may be any where in the plane $A'B'C'D'$; Since a line which

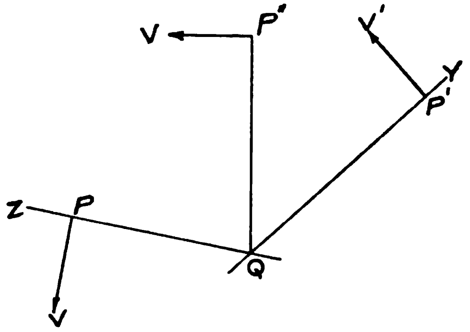


Fig. 12.

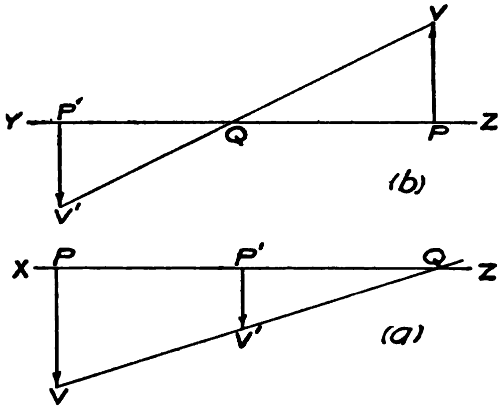


Fig. 13.

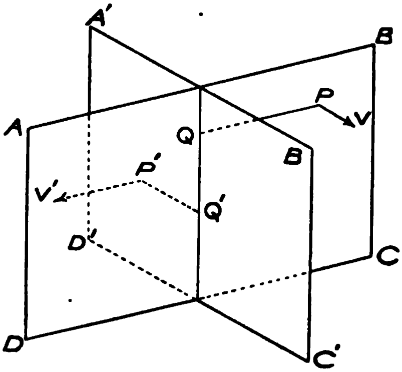


Fig. 14.

lies in two planes must lie at their intersection, the line passing QQ' is the instant axis sought.

29. Axes of Rotation of Rigid Bodies.—The motion of a rigid body may be referred to an axis which intersects the body as in the case of the fly-wheel of Fig. 1, where the axis is at the center of the wheel or the motion of a body may be referred to an axis which lies entirely without the body as that of the fly balls on the governor in the same figure.

If an *instant axis* of rotation remains constantly in one position it is called a *fixed axis.* These axes may be located by drawing a line through any point on the perpendicular to the direction of motion at the instant considered.

If the motion of two or more bodies is considered, their simultaneous axis will pass through the intersection of the perpendiculars to their instantaneous directions of motion. This axis can always be located when the motion of two of the bodies is known. Then the motion of any other body in the system can be found by drawing a line through the axis and the center of the body and taking its direction perpendicular to this line.

A case of common occurrence in the analysis of mechanism is that of two bodies having relative motion to each other and also to a third body which may be either fixed or movable. Besides the simultaneous instant axes already noted these bodies will each have an axis through the third body about which it rotates and they will both contain a point that has the same linear velocity in each of them. It can be proved that these three points always lie in the same straight line and by taking advantage of this property it is often possible to determine the direction and velocity of parts of a complicated piece of mechanism from the known properties of other parts.

30. Application of the Instant Center and Velocity Diagram.—In Fig. 15 which may be taken to represent the crank, connecting rod and cross head of the steam engine of Fig. 1, suppose the velocity of C is required at a given instant, the velocity of P being known. The instant center of C lies on the perpendicular to the path of C. The instant center of P lies on the perpendicular to PT, which is tangent to the path of P. Their common instant center will lie at Q and through Q will pass the instant axis of the rod. If the linear velocity of P is PN, draw QN and then lay off on QC continued the distance QP' equal to QP. At P' lay off the perpendicular $P'N'$ equal to PN and draw QN'. Then CM will be the velocity of C at the instant taken, for in the triangles $QP'N'$ and QCM, $CM:P'N'::QC:QP'$. Hence CM is to $P'N'$ as the velocity of C is to the velocity of P. The velocities being proportional to the distances from Q.

31. Two Axes and Point of Common Velocity.—In Fig. 16, let A and B be two wheels having axes O and Q and a point of common velocity P. Then these three points will lie in the same straight line and if the velocity of P is known the velocity of any point in either wheel can be determined.

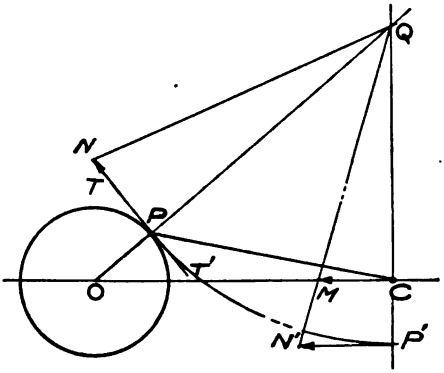


FIG. 15.

Suppose P to lie at P' anywhere off of the line OQ. The velocity and direction of P around O at any instant is $P'N$, perpendicular to OP'. The velocity and direction of P around Q at the same instant will be $P'N'$, perpendicular to QP'. Hence we have a point moving in two directions at the same instant with the same velocity, which is impossible. P can have the same linear velocity about both O and Q only when its direction about both points is the same. Since the directions are always perpendicular to the radii, if the perpendiculars coincide in direction the radii must lie in the same straight line, and this happens only when they coincide with the line OQ, or when O, P and Q are in the same straight line.

If PV represents the instant velocity of P then LM, the velocity of L, any other point on A, is found by laying off its distance from O an OP, and constructing the triangle OLM, similar to OPV. In like manner the instant velocity of R, any other point on B, may be found.

32. Axes and Velocity Points in a System of Bodies.—By using the principle of Art. 31, the axes and velocity points in a train of mechanism that contains more than two bodies may be found. In analyzing the motions of such a train the bodies are taken in pairs and their motion considered relative to a third body which may be either fixed or movable.

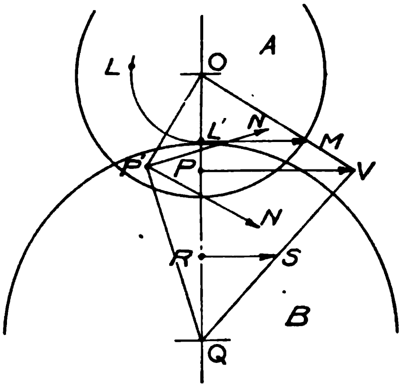


FIG. 16.

In the four-link mechanism, Fig. 17, let B, C and D be movable bodies and let A be fixed. Each of the moving bodies will have an axis through A about which it rotates and each pair of moving bodies will have a point of common velocity.

The axis in A about which B rotates is ab, that of D is ad. The point of common velocity on D and C will be cd, and the point of common velocity between B and C will be bc.

To find the fixed axis in A, or A extended, about which C rotates, take the direction of cd and cb at any instant and find the intersection of the perpendiculars to these directions, as at ac. To find the point of common velocity on B and D; since the axes and point of common velocity of two bodies lie on the same straight line, the point sought must lie somewhere on the line *ad*—*ab*, or on an extension of it. The axes on *C* about which *D* and *B* rotate are *cd* and *bc* and the point of common velocity will lie on a line through these points, hence it must lie at the intersection *bd*.

Tabulating the bodies with their centers and velocity points we have the following arrangement:

Bodies	*A*	*B*	*C*	*D*
Axes and velocity points	*ab*	*bc*	*cd*	
	ac	*bd*		
	ad			

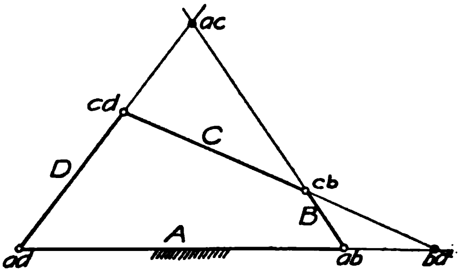


FIG. 17.

Of these *bd*, *bc* and *cd* are points of common velocity.

Since there are four links and the total number of points involved is six the number of points and axes may be expressed by the formula:

$\frac{N(N-1)}{2}=P$, where N is the number of bodies in the system and P is the total number of points.

The number of velocity points is equal to the number of pairs of moving bodies, in this case three.

33. Analysis of a Mechanism of Six Bodies.—In Fig. 18 there is a fixed body *A*, and five moving bodies *B*, *C*, *D*, *E*, and *F*. The total number of axes and velocity points will be,

$$\frac{6(6-1)}{2}=15.$$

They will be arranged in the following order:

A	*B*	*C*	*D*	*E*	*F*
ab	*bc*	*cd*	*de*	*ef*	
ac	*bd*	*ce*	*df*		
ad	*be*	*cf*			
ae	*bf*				
af					

The figure shows the location of all the points. All of the intersections which involve *A* are fixed axes and there are five of them.

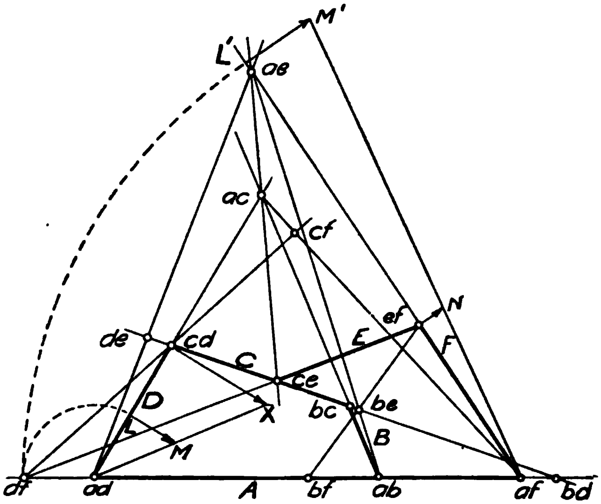


Fig. 18.

It must be noted in the solution of problems of this class that an axis in a body may lie either in the body as shown or in an extension of it. Thus the axes *ac* and *ae* are in an extension of *A*.

There are ten possible arrangements of the moving bodies in pairs and consequently ten points of common velocity.

The location of the axes and points is found as follows:

Take any two of the bodies as *B–D* and consider their motion with respect to some third body *A*. The axes of *D* and *B* in *A*, are *ad* and *ab*, the point of common velocity must therefore lie on this line or an extension of it. Now take the motion of *B* and *D* with relation to some other third body as *C*. The axes of *D* and *B*, in *C*, are *cd* and *bc*. The point of common velocity must therefore lie on this line or an extension of it. The intersection of the two lines gives *bd*, the point sought.

The use of the letters to designate the axes and velocity points in the manner here shown affords a convenient check on the work for it will be seen that the designation of the three points contains only the three letters which name the bodies involved Thus in the points *ab*, *ad* and *bd* we have only the letters *A*, *B* and *D* which are the names of the bodies under consideration.

Suppose it is required to find the velocity of any point as *ef* in this mechanism when the velocity of some other point, as *cd*, is known. Lay out *cd–X*, perpendicular to *cd–ad*. Then by similar triangles, the velocity of *df* in *D* will be found to equal *LM*. The body *F* rotates about *af* and *df* has the same velocity in both *F* and *D*. Lay off *L'M'* equal to *LM* on *F* extended and as far from its center of rotation as that center is from the point of common velocity *df*. Then by similar triangles, the velocity of *ef* is found to be *ef–N*.

From this the following rule may be deduced for finding the velocity of any point in a mechanism when the velocity of some other point is given: *From the velocity of the given point find the velocity of the common point, then from the velocity of the common point find the velocity of the required point.*

PROBLEMS

1. Locate the centrode of a point moving in the circumference of a circle.

2. Locate the instant center of a connecting rod of an engine when the rod is 18 in. long, the crank 6 in. long and the center line of the rod tangent to the crank pin circle.

3. Locate an instant axis of the three moving points *A*, *B* and *C*; *A* moving in a straight line, *B* in the circumference of a circle and *C* in an irregular curve, the plane of all the motions being parallel.

4. *AB*, *AC*, *AD* and *AE* are four motions having velocities of 10, 20, and 37 ft. per minute respectively. If *AB* is horizontal and the other motions make angles of 10, 15 and 30 degrees respectively in the same plane with it, find the resultant motion.

5. A body is moving at the rate of 30 ft. per second. (a) Find its two components, one making an angle of 15 and the other 30 degrees with it.

(b) The same motion has one component of 10 ft. per second making an angle of 15 degrees with it. Find a second component.

(c) Same motion has components of 25 and 12 ft. per second. Find their direction.

6. Two circular disks, of 10 in. and 20 in. diameter, are in contact and the larger makes 100 r.p.m. Find the linear velocity at the point of contact and the linear velocity of a point 5 in. from the center of the large disk.

7. In Fig. 17, let the links have the following dimensions: *A* 5 in., *C* 4 in. *B* 1 in. and *D* 2 in. Find the velocity of the point *bc*, when *D* is rotating at the rate of 50 r.p.m.

8. Find all the axes and velocity points and the velocity of all the points under the following conditions in a mechanism similar to:

Length of *A* 6 in.
Length of *C* 4 in.
Length of *E* 2 in.
Length of *F* 3 in.
Length of *B* 1 in.
Length of *D* 1½ in.
Distance from *ab* to *af* 2 in.
Distance from *ec* to *be* 1½in.

CHAPTER III

FUNDAMENTAL RULES IN TRANSMITTING MOTION FROM A DRIVER TO A FOLLOWER

34. Directional Relation.—In Fig. 19, *A* and *B* may be any two curved pieces, in contact at a particular instant at *P*. *P* is called the point of contact, *OP* and *O'P* are the radii of contact, *TT'* drawn through *P* tangent to the curves is called the common tangent, *NN'* and a line through *P* perpendicular to the tangent, is the common normal.

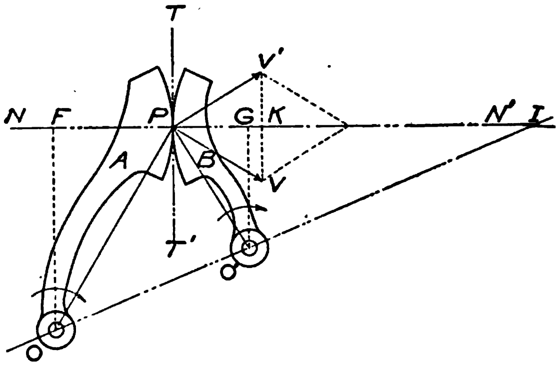


Fig. 19.

If *A* is turning about *O* as a center, the point of contact on *A* will describe a circle about *O*, with radius *OP*. Assuming *A* to rotate clockwise, the direction of *P* at the instant taken will be *PV*, tangent to the circle that *P* describes about *O*, and perpendicular to the radius *OP*. Also, if *B* is turning about *O'* the point of contact on *B*, will have the instantaneous direction *PV'*, perpendicular to *O'P*. Then *B* also turns clockwise, as shown by the figure, otherwise the pieces would pass or intersect each other, which is contrary to the assumption that they are to remain in contact with one piece driving the other.

In Fig. 20 the fixed centers O and O' are on opposite sides of the common normal. Suppose A to be the driver and to rotate clockwise, then the direction of the point of contact on A at a given instant can again be represented by PV. If the pieces do not pass or intersect each other the motion of the point of contact on B, at the same instant will be represented by PV'. Then the rotation of B is counter-clockwise, and the two pieces turn in opposite directions.

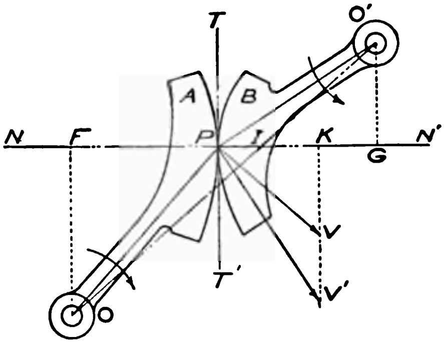


FIG. 20.

Therefore for two pieces in direct contact:

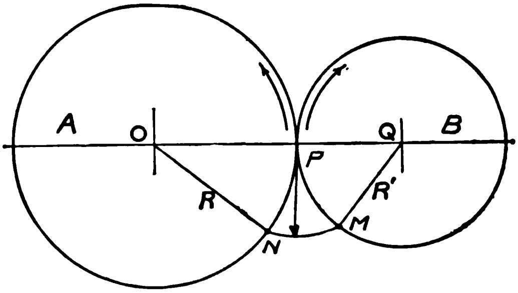


FIG. 21.

(a) *If the fixed centers of motion are both on the same side of the common normal, the pieces will rotate in the same direction, and if the fixed centers are on opposite sides of the common normal, the pieces will rotate in opposite directions.*

The application of this law to circular wheels is shown by Fig. 21. Here the point of contact lies between the centers, and the wheels rotate in opposite directions. In Fig. 22 the centers are both on the same side of the point of contact and the wheels will rotate in the same direction.

In Fig. 3 was shown a mechanism in which the relative position of the centers of driver and follower changes as motion proceeds and the wheels rotate at first in the same direction and then in opposite directions.

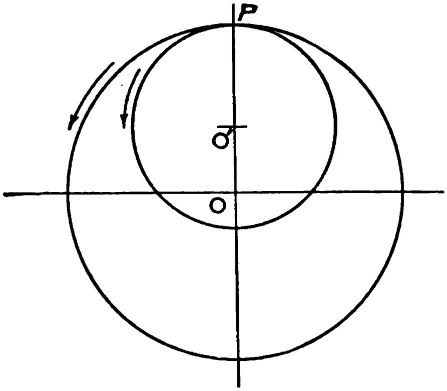


Fig. 22.

35. Velocity of a Follower in Direct Contact.—Motion is transmitted by direct contact, by virtue of the normal pressure between the acting surfaces and the velocity of two pieces in the direction of the common normal must be equal or the pieces will either separate or intersect. Then if PV, Figs. 19 or 20, represents the linear velocity of a point on A at any instant, the linear velocity of a point on B at the same instant can be found, by aid of the triangle of velocities as follows: Through V draw a perpendicular to the common normal, and produce it until it intersects the perpendicular to $O'P$ at V'. In the triangle $PV'K$, PK is known in amount and direction, being the base of the known triangle PVK, and having the direction of the common normal; also the other two sides PV' and $V'K$ are known in direction. Then PV' which represents the velocity of B, is completely determined. Hence if the direction is known, when two pieces are moving in direct contact:

(b) *If the velocity of a point on one piece, at a given instant, is* *known, the velocity of a corresponding point on the other piece can be found, because the components of these velocities in the line of the common normal are equal.*

36. Velocity Ratio.—If the angular velocity of a driver be divided by the angular velocity of the follower, the quotient will be the *velocity ratio.* This ratio may be either constant or varying according to the shape of the bodies in contact. Thus two circular wheels will have constant velocity ratio while two elliptical wheels will have a constantly varying velocity ratio.

In Figs. 19 and 20, let ω_1 equal the angular velocity of A about O, and ω_2 the angular velocity of B about O'.

The angular velocity of either piece $=\dfrac{\text{linear velocity of } P}{\text{radius of contact}}$.

Also, as has just been shown, the linear velocity of a point on either piece can be found if the velocity of a corresponding point on one of them is given.

Dividing the linear velocities by their respective radii will give the angular velocities, and finally the angular velocity *ratio* can be determined by dividing one angular velocity by the other. A more direct method of finding this ratio is given by the following: drop the perpendiculars OF and $O'G$ from the fixed centers upon the line of the common normal.

$$\omega_1=\frac{PV}{PO} \text{ and } \omega_2=\frac{PV'}{PO'}$$

The triangles PVK and POF are similar;

Also, triangles $PV'K$ and $PO'G$ are similar.

$$\therefore \frac{PK}{OF}=\frac{PV}{PO} \text{ and } \frac{PK}{O'G}=\frac{PV'}{PO'}$$

Then
$$\frac{\omega_1}{\omega_2}=\frac{PK}{OF}\times\frac{O'G}{PK}=\frac{O'G}{OF}$$

Let the line of centers intersect the common normal at I. The triangles IOF and $IO'G$ are similar,

and
$$\frac{IO'}{IO}=\frac{O'G}{OF}=\frac{\omega_1}{\omega_2}$$

Hence for two pieces moving in contact:

(c) *The angular velocity ratio is inversely proportional to the* *perpendiculars let fall from the fixed centers of motion upon the common normal, or the angular velocity ratio is inversely proportional to the segments into which the line of centers is divided by the common normal.*

The application of this law to a pair of circular wheels is shown by Fig. 21.

Let N and M be two points having the same linear velocity, then angular velocity of $A = \frac{N}{R} = \omega_1$

and,

angular velocity of $B = \frac{M}{R'} = \omega_2$

Then $$\frac{\omega_1}{\omega_2} = \frac{\frac{N}{R}}{\frac{M}{R'}} = \frac{R}{R'}$$

37. Directional Relation, Velocity and Velocity Ratio, Link Connectors.—In Figs. 23 and 24, let O and O' be fixed centers, and A and B rotating arms connected by the link PP'.

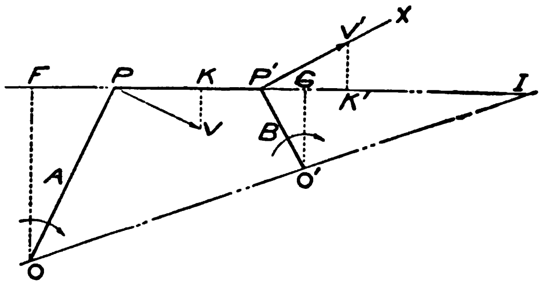


FIG. 23.

In Fig. 23, if A is the driving arm and P moves at a given instant in the direction PV perpendicular to OP, then P' must move in the direction $P'V'$ perpendicular to $O'P'$, or the link would be shortened. But the assumed rotation of A is clockwise, therefore from the figure the rotation of B is also clockwise, and the arms turn in the same direction. In Fig. 24, where O and O' are on opposite sides of the link, if the motion of P at a given instant is represented by PV, then P' must move in the direction $P'V'$, or the link would be stretched. But here the direction of A is counter-clockwise, and the direction of B is clockwise, therefore in this case A and B turn in opposite directions.

Again, in either figure, if PV represents the linear velocity of P at any instant, the component of this velocity in the line of the link will be PK, and this component must equal the component of $P'V'$ in the line of the link, or else the points P and P' would either approach each other or separate, which is contrary to the idea of a rigid link. Then if PV is given, $P'V'$ can be found as follows: Drop the perpendicular VK and lay off $P'K'$ equal to PK. Draw $P'X$ perpendicular to $O'P'$. Erect a perpendicular at K' intersecting $P'X$ in V'.

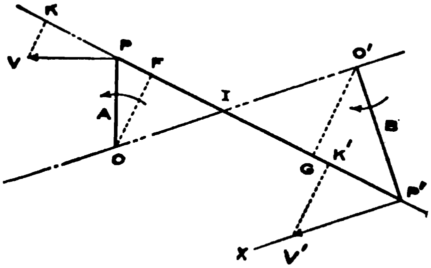


FIG. 24.

To find the angular velocity ratio of the pieces, drop the perpendiculars OF and $O'G$.

Remembering that angular velocity $= \dfrac{\text{linear velocity}}{\text{radius}}$:

let $\omega_1 =$ angular velocity of $A = \dfrac{PV}{PQ}$

and $\omega_2 =$ angular velocity of $B = \dfrac{P'V'}{P'O'}$

The triangles PVK and POF are similar:
Also triangles $P'V'K'$ and $P'O'G$ are similar.

Then $$\frac{PK}{OF} = \frac{PV}{PO} = \omega_1$$

And $$\frac{P'K'}{O'G} = \frac{P'V'}{P'O'} = \omega_2$$

Therefore $$\frac{\omega_1}{\omega_2}=\frac{PK}{OF}\times\frac{O'G}{P'K'}=\frac{O'G}{OF}.$$

(Since $PK=P'K'$)

Let the line of centers intersect the line of the link at I.

The triangles IOF and $IO'G$ are similar.

Therefore $$\frac{IO'}{IO}=\frac{O'G}{OF}=\frac{\omega_1}{\omega_2}.$$

Conclusions similar to (a), (b), and (c) of Arts. 34, 35 and 36 can now be derived for a pair of rotating arms connected by a rigid link:

(d) *If the fixed centers of motion are both on the same side of the line of the link, the arms will rotate in the same direction; but if they are on opposite sides of the line of the link, the arms will rotate in opposite directions.*

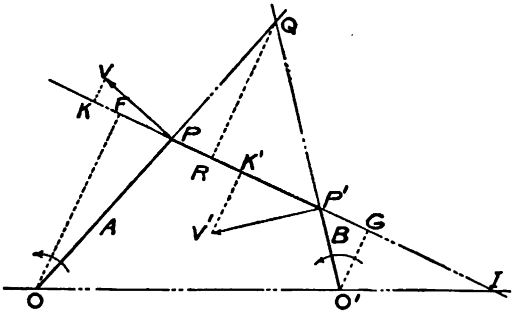


FIG. 25.

(e) *If the instant velocity of any point in the link is known, the velocity of any other point can be found, for the components of these velocities in the line of the link are equal.*

(f) *The angular velocity ratio of the arms is inversely proportional to the perpendiculars let fall from the fixed centers of motion upon the line of the link, or it is inversely proportional to the segments into which the line of centers is divided by the line of the link.*

38. Angular Velocity Ratio of Links by Instant Centers.—In Figs. 25 and 26 produce OP and $O'P'$ until they intersect at Q. Then Q is an instant center for the motions of P and P' and hence for the link PP'.

Suppose ω to represent the instantaneous angular velocity of PP' about Q.

Let PV be the linear velocity of P at a given instant, and $P'V'$ the linear velocity of P' at the same instant.

$$\text{Then } \omega = \frac{PV}{PQ} = \frac{P'V'}{P'Q}.$$

Drop the perpendiculars VK, OF, $O'G$, $V'K'$ and QR upon the line of the link.

The triangles QPR and OPF are similar; also the triangles $QP'R$ and $O'P'G$ are similar.

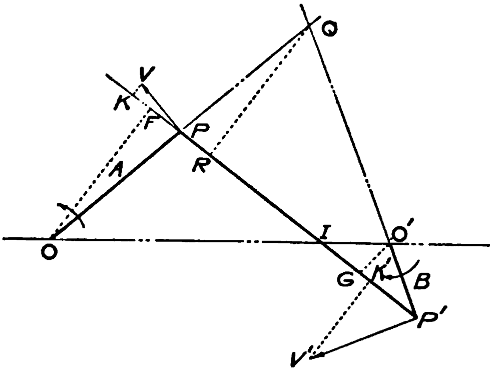


FIG. 26.

Then if ω_1 and ω_2 represent the angular velocities of P and P' about the centers O and O':

$$\frac{\omega_1}{\omega} = \frac{PV}{OP} \times \frac{PQ}{PV} = \frac{PQ}{OP} = \frac{QR}{OF} \tag{1}$$

$$\frac{\omega}{\omega_2} = \frac{P'V'}{P'Q} \times \frac{O'P'}{P'V'} = \frac{O'P'}{P'Q} = \frac{O'G}{QR} \tag{2}$$

From (1) and (2) we have $\frac{\omega_1}{\omega} \times \frac{\omega}{\omega_2} = \frac{\omega_1}{\omega_2}$ or $\frac{QR}{OF} \times \frac{O'G}{QR} = \frac{O'G}{OF} = \frac{IO'}{IO'}$ which is equivalent to statement (*f*) Art. 37.

39. Directional Relation Velocity and Velocity Ratio, Wrapping Connectors.—In Figs. 27 and 28, let A and B be two curved pieces, connected by a flexible and inextensible band PP'. As the pieces rotate about the centers O and O', assuming A to be the driver and the rotation to take place in the direction shown, the band will be unwrapped from B and wrapped around A. If only that part of the band between the tangent points P and P' is considered, it will correspond to the link PP' in Figs. 23 and 24, and the radii of contact OP and $O'P'$ will correspond to the arms OP and $O'P'$. Then by drawing PV and $P'V'$ to represent the instant velocities, dropping the perpendiculars OF and $O'G$, and locating I, the latter being the intersection of the line of centers with the line of the belt, statements like (d), (e), and (f) of Art. 37 can be shown to hold true for a wrapping connector. By producing the radii of contact OP and $O'P'$ until they intersect,

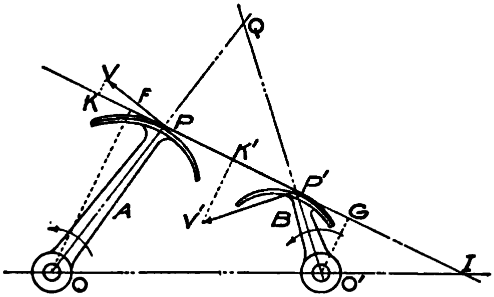


Fig. 27.

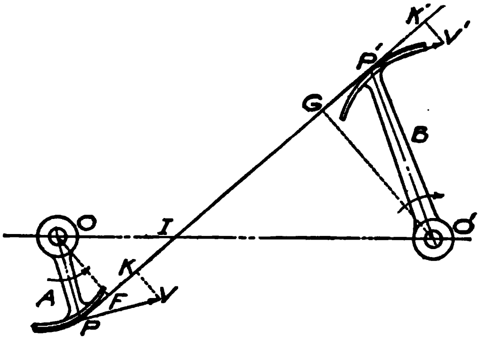


Fig. 28.

the velocity ratio may be determined by the method of instant centers, as in Art. 38.

Therefore, for rotating pieces connected by a flexible band:

(g) *If the fixed centers of motion are both on the same side of the wrapping connector, the two pieces will rotate in the same direction; and if the centers are on opposite sides of the wrapping connector, the pieces will rotate in opposite directions.*

(h) *If the velocity of a point on the driver is known, the velocity of the corresponding point on the follower can be found, because the components of these velocities in the line of the connector are equal.*

(i) *The angular velocity ratio of the pieces is inversely proportional to the perpendiculars let fall from the fixed centers of motion upon the line of the wrapping connector, or it is inversely proportional to the segments into which the line of centers is divided by the line of the wrapping connector.*

40. The General Case.—In each of the preceding cases if the equivalent expression, *line of action,* be substituted for "common normal," "line of the link," and "line of the wrapping connector," then three perfectly general statements can be made regarding the velocity and directional relations of moving pieces.

I. *If the fixed centers of motion are both on the same side of the line of action, the driver and follower move in the same direction, but if the centers are on opposite sides of the line of action, the pieces will move in opposite directions.*

II. *The components, in the direction of the line of action, of the instant velocities of a driver and follower are equal and if the velocity of one element is given, the other can be found.*

III. *The velocity ratio of driver to follower is inversely proportional to the perpendiculars let fall from the fixed centers of motion upon the line of action, or it is inversely proportional to the segments into which the line of centers is divided by the line of action.*

These rules are true, no matter which piece is taken as the driver or follower.

41. Condition for Constant Angular Velocity Ratio.—It is often necessary to have the velocity ratio between the driver and follower a constant. From rule III of Art. 40, Figs. 21 to 28, we have:

$$\frac{\omega_1}{\omega_2}=\frac{IO'}{IO},$$

Now if $\frac{\omega_1}{\omega_2}$ is to be constant, then $\frac{IO'}{IO}$ must be constant that is, the two segments of the line of centers must be of constant length. Therefore, the condition for constant angular velocity ratio:

IV. *The line of action must cut the line of centers in a fixed point.*

42. Definition of Rolling and Sliding Contact.—When one piece of mechanism is driving another by direct contact, the nature of the action between them is either *pure rolling, pure sliding*, or *mixed* rolling and sliding.

In pure rolling contact, consecutive points on the driver come in contact with consecutive points on the follower, as the motion proceeds. In Fig. 29, if P, 1, 2, 3, are consecutive points on A, and P, 1′, 2′, 3′, are consecutive points on B, then as the two pieces turn about the axes O and O', 1 will fall at 1′, 2 will fall at 2′, and so on, as these points come successively to the line of centers.

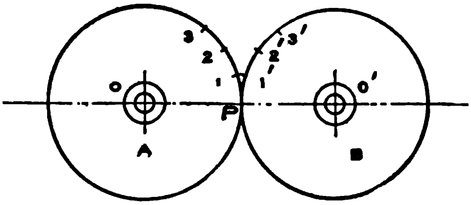


FIG. 29.

In pure sliding contact, a single point on one element must come in contact with all the consecutive points on the other. That is, if B is stationary and A turns, the point P, on B, will come in contact with all the points 1, 2, 3, on A.

If both pieces turn, but A turns more slowly than B, so that there is still slipping between the two surfaces, the contact is mixed rolling and sliding.

43. Condition for Pure Rolling Contact.—In Fig. 30 let A drive B by direct contact and let PV and PV' represent the velocity of a contact point on each piece at a given instant. The normal component PH shows the velocity with which motion is positively transmitted or the velocity with which A pushes B, and the tangential components PK and PG indicate the lost motion, or that part of the velocity which is not being transmitted. When the fixed centers are on opposite sides of the common normal, as in Fig. 30, [*PG*−*PK*] represents the velocity of sliding between the pieces. If the centers are on the same side of the common normal, as in Fig. 31, the tangential components *PG* and *PK* lie in opposite directions and their sum equals the velocity of sliding. In pure rolling contact, the sliding of one piece on the other is zero, that is, the tangential components are both zero or equal. By Art. 35 (b) the normal components are always equal. Now *both* the normal and tangential components of *PV* and *PV′* are equal only when *PV* and *PV′* are equal and coincide in direction. But *PV* is perpendicular to *OP*, and *PV′* is perpendicular to *O′P*. Then if *PV* and *PV′* lie in the same direction, *OP* and *O′P* must lie in the same direction, and *P* will be located on the line of centers. Hence the condition for pure rolling contact:

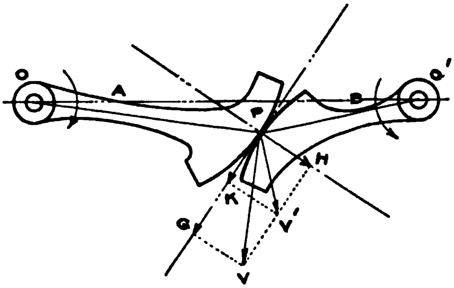


Fig. 30.

V. *The point of contact must lie on the line of centers.*

44. Constant Angular Velocity Ratio and Pure Rolling Combined.—There are a number of curved surfaces which will act together and fulfil the condition of constant angular velocity ratio or the condition of pure rolling contact, but rolling circles are the only curves that will fulfil both conditions at the same time. The driving action between a pair of rolling circles, however, is not positive, for motion is transmitted from one to the other by virtue of the friction between the two acting surfaces.

45. Condition for Positive Driving.—When one piece is driving another by direct contact, the action is positive if the point of contact on the driver displaces the point of contact on the follower along the normal. In Fig. 31 the driving action is positive, and the figure shows that if the pieces are to remain in contact, the contact radius of the driver must increase as fast as that of the follower decreases. This is sometimes stated as the condition of positive driving, but the statement is not perfectly general; for example, in Fig. 32, *A* may rotate and drive *B* continuously, while its radius remains constant. Again in Fig. 33, the fixed centers might be so located that, with either *A* or *B* as a driver, the radius of one would increase as the other decreased for a short duration of the motion; but when a position such as that shown in the figure is reached, there will be no

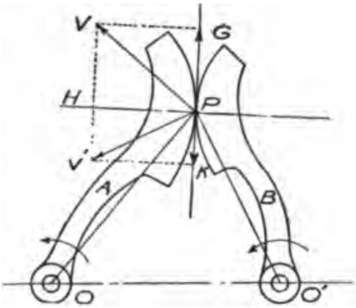


FIG. 31.

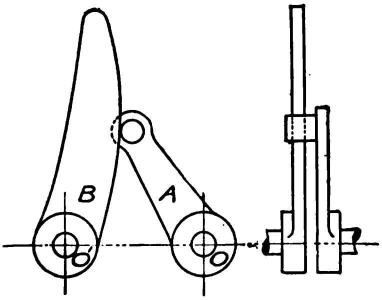


FIG. 32.

motion transmitted, for the normal to the acting surfaces passes through O', and the effect of A as a driver is only to compress B or to move the center O'. Whenever positive driving does take place, the common normal does not coincide with the line of centers. This is shown by Figs. 19, 20, 30 and 31.

The following statement, therefore, of the condition of positive driving is general and true for all cases of contact motion:

VI. *The common normal must not pass through either of the fixed centers.*

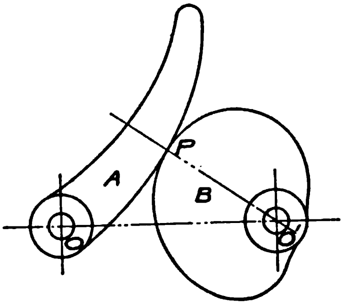


FIG. 33.

46. From the above conditions of constant velocity ratio, pure rolling contact and positive driving, it follows that any two of the conditions may occur simultaneously, in a pair of contact pieces, but that all three conditions will never, except in the single instance noted later, be present at the same time.

PROBLEMS

1. Prove that two rotating cylinders in external contact will revolve in opposite directions.

2. Two curved pieces with fixed centers 8 in. apart are in external contact. The radius of one is 5 in. and its velocity at a given instant is 2π radians per second. Find the velocity of the other piece.

3. Prove that the velocity ratio in Prob. 2 is inversely proportional to the perpendiculars let fall from the fixed centers of motion upon the line of action.

4. Prove that the velocity ratio in Prob. 2 is inversely proportional to the segments into which the line of centers is divided by the line of action.

5. Prove that the angular velocity ratio of a pair of non-circular wheels is not constant.

6. Prove that a locomotive wheel rolling on a level track is theoretically in pure rolling contact.

7. Show why a steam engine will not start when the crank is on its dead center.

8. If two curved pieces are moving in external contact, prove that the action is pure rolling when the tangential components of their velocities are zero.

9. Show why a driver cannot simultaneously transmit positive motion to a follower by pure rolling contact and with a constant velocity ratio.

10. A pulley turning at the rate of 125 r.p.m. drives another at the rate of 375 r.p.m. What is the velocity ratio?

11. Assuming that the first pulley in example 11 is 36 in. in diameter, how large is the second?

12. Represent graphically the speed and directions, at a given instant, of a point moving in the circumference of a 12-ft. circle at the rate of 100 r.p.m.

CHAPTER IV

LINK CONNECTORS

47. A link is a rigid connector joining two other mechanical elements. It is known under a variety of names such as side rod, connecting rod and pitman. The connected pieces are called cranks when they make complete rotations and crank arms, beams, rocker arms and levers when they oscillate.

A link is either simple or compound, depending upon the number of points of its attachment. A simple or binary link has two points of attachment and may be represented by a straight line with a point of attachment at each end. A compound link having three points of attachment, represented by a triangle with the points at the corners, is a ternary link. A compound link in the form of a rectangle with the points of attachment at each corner is a quarternary link and so on.

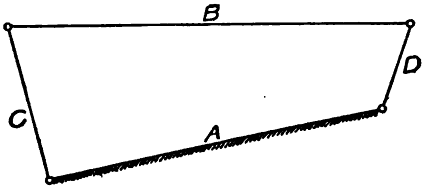


FIG. 34.

When a number of links are joined together by their points of attachment in such manner that a definite motion of one of them causes a definite motion of the others the arrangement is a kinematic chain or a *closed* kinematic chain. Thus Fig. 34 is a four-link, closed kinematic chain and constitutes the simplest-case of the complete linkwork chain. If the number of links be reduced to three, they would form a triangle and no motion could be transmitted from one link to another, again if the number be increased to five the motion of one of them would be indefinite and uncontrolled by that of the others.

If L equals the number of links and P equals the number of joints or points of attachment the relation between these links and joints in a closed chain will be,

$$P=\frac{3}{2}L-2$$

It follows from this that if a closed chain have more than four links one of them must be compound.

Kinematic chains are used:

I. To transform continuous rotation into rectilinear reciprocating motion or the reverse.

II. To transform continuous rotation into rotative reciprocating motion or the reverse.

III. To transform rotative reciprocating motion into rectilinear reciprocating motion or the reverse.

IV. To transmit rotative reciprocating motion.

V. To transmit continuous rotation.

In classes II, IV and V the four-link chain is used and in I and III the slider crank mechanism is employed.

48. Class I. Rotation to Rectilinear Reciprocating Motion.—The slider crank mechanism, Fig. 35, illustrates the relative motion of the crank, connecting rod and cross head of a steam engine, the circular motion of the pin P being changed to the rectilinear reciprocating motion of the block K. As P moves from A to B in the direction indicated, K moves from D to C and as P moves from B to A, K returns from C to D, CD being equal to twice OP. The points A and B are called *dead points* because when P is at either of these points, the arm OP and the link PK lie in the same straight line and a force applied at K has no tend-

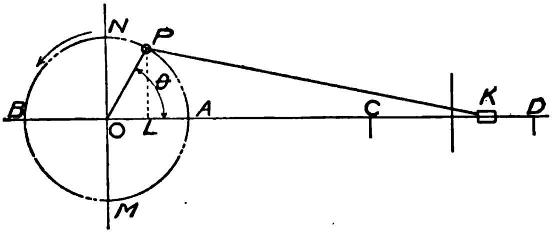


FIG. 35.

ency to move the arm OP. A may be termed the *inward* dead point because P, when in that position, lies between K and the center O. B is the *outward* dead point. C and D are called stopping points because K comes to rest and changes direction there.

Two problems present themselves in studying a linkage of this form:

(a) To find the position of K with respect to O, for any given position of P.

(b) To find the linear velocity ratio of K to P, when the arm is in a given position.

49. Relative Position of Pin and Block.—In Fig. 35 the link $PK=AD=BC$.

Draw PL perpendicular to the center line AB. Then by trigonometry,

$$OL=OP\cos\theta \text{ and } PL=OP\sin\theta.$$

By geometry, the distance,

$$LK=\sqrt{PK^2-PL^2}=\sqrt{PK^2-(OP\sin\theta)^2}.$$

But from the figure,

$$OK=LK+OL.$$

Then by substituting values of OL and LK we have,

$$OK=\sqrt{PK^2-(OP\sin\theta)^2}+OP\cos\theta,$$

from which, with any position of P known, K can be located by measurement.

When the value of θ is between 90° and 270°,

$$OK=LK-OL,$$

and $\cos\theta$ is negative.

If the link PK were infinitely long then the distance from D to K at any instant would equal the distance from A to L, but owing to the angularity of the link, when P is at N the block K will have moved over more than half of the distance from D to C. On the return stroke, when P reaches M, K will not yet have reached its mid position.

The eccentricity of K increases as the length of the arm approaches that of the link.

50. Velocity Ratio of Pin and Block.—If the link PK were infinitely long, the motion of K would be like that of L, and its maximum velocity would occur at the instant when P is at N. Due to the angularity of the link this maximum velocity of K occurs when the link is tangent to the path of P.

In Fig. 36, if x represents the velocity of P and y represents the velocity of K, then by Art. 37 (e) the components of these velocities in the line of the link are equal, that is $CP=FK$. Then by trigonometry

$$x \cos APC = y \cos BKF. \tag{1}$$

Draw ON perpendicular to OK, and OD perpendicular to PK produced.

The angle $POD = APC$,
and $DON = BKF$.

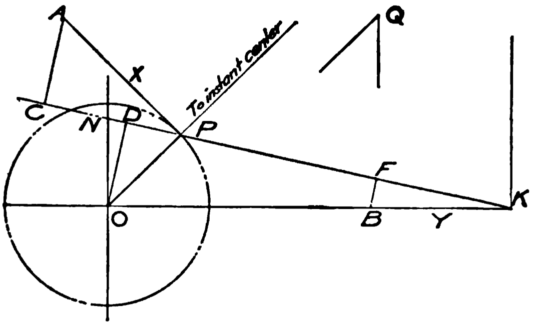


FIG. 36.

Then $$\cos APC = \cos POD = \frac{OD}{OP} \tag{2}$$

and $$\cos BKF = \cos DON = \frac{OD}{ON} \tag{3}$$

Substituting values from (2) and (3) in equation (1) we have,

$$x\frac{OD}{OP} = y\frac{OD}{ON}$$

Then $$\frac{x}{y} = \frac{OP}{ON}, \text{ a variable.}$$

When $ON = OP$, the velocities of P and K are equal. This occurs four times in each revolution, twice when OP is perpendicular to OK and twice when P is in such position to right of ON that PON is an isosceles triangle.

This ratio may also be found by locating the instant center Q, as in Fig. 15, and dividing the velocity of the driver thus found by that of the follower.

51. Velocity Diagram.—The velocity and acceleration of the sliding block, either at a given instant or at a given point in its path may be conveniently represented by a diagram.

In Fig. 37 divide ANB, the path of P, into any number of equal parts. With radius equal to the length of the link PK, and with centers at 1, 2, 3, ... 7, intersect $A'B'$, the path of K, in points 1′, 2′, 3′, ... 7′. Erect perpendiculars at these points and draw lines through 1′–1, 2′–2, ... 7′–7, and produce them when necessary to intersect the perpendicular ON.

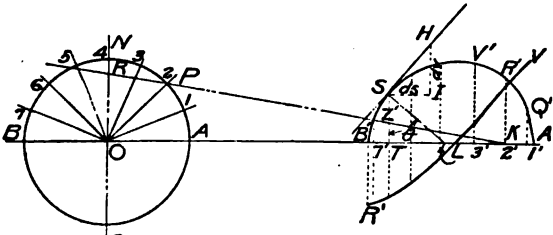


FIG. 37.

In the figure, the line 2–2′ intersects ON at R, then by Art. 50, OR represents the velocity of K when P is at 2. Lay out this velocity at $2'$–R' and in the same way find the velocities of K for the other positions of P and lay them out at $1'$–Q', $3'$–V', ... $7'$–Z'. A smooth curve through the points Q', R', Z', will be the required diagram.

If the time required by P to travel from A to B be laid off to any scale and ordinates be erected at the division points as just described, the resulting figure will be a velocity time diagram.

If at any point S, on the velocity curve, a tangent and a normal be drawn to the curve, the intercept LT on the line of centers will represent the acceleration of K when at the point T. For, if s represents the space passed over in any time t, then v, the velocity, $=\frac{ds}{dt}$, or $ds=vdt$, also the acceleration, $a=\frac{dv}{dt}$.

In the figure, $\tan\theta=\frac{dv}{ds}=\frac{dv}{vdt}=\frac{1}{v}\times\frac{dv}{dt}=\frac{a}{v}$.

The triangles SHI and STL being similar, angle $ISH=$ angle $LST=\theta$.

Then, $\tan\theta=\frac{TL}{TS}=\frac{a}{v}$, and since $v=TS$, a must equal TL.

The acceleration of K at the other points of its path may be found in the same way and the curve RV found by laying off the acceleration on the corresponding ordinate.

The tangent changes sign near the mid-position of K and the curve RV lies above the line of centers on one side of the mid-position and below the line of centers on the other, which shows that the acceleration of K is first positive and then negative. This method of constructing the acceleration curve is not accurate, since it depends upon drawing a tangent to an irregular curve. The acceleration of K at any point in its path may also be calculated by the following formula

$$A=\frac{Wv^2}{gR}\left(\cos\theta\pm\frac{\cos.2\theta}{n}\right)$$

where W is the weight of K and its connections,

v = its velocity,
R = the radius OP.
g = 32.2
θ = the angle AOP.
$n=\frac{PK}{OP}$

This acceleration may then be laid out as an ordinate in the diagram and furnish a check on the graphical method.

52. Examples of Class I Motions. **Eccentrics.**—An important application of the links of Class I is made in driving the slide valves of steam engines. A circular disk, Fig. 38, with center at P, is fastened to a shaft having a center at O. A band F surrounds the disk and is rigidly fixed to the link HK. As the disk turns about O it slides within the band and transmits the same motion to K as would be given by a small crank OP turning about the center O. This is virtually a crank having a pin of larger radius than the crank. The throw of an accentric is sometimes given as OP and sometimes as $2\,OP$. On steam engines K is often connected to one end of an oscillating lever, in which case the mechanism is an example of Class II.

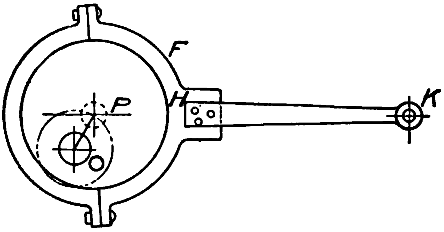


Fig. 38.

53. Crank and Link Equal.—If the length of a link and crank are made the same, as in Fig. 39, then K will be drawn up to O when P reaches N and the acceleration of K due to the angularity of the link will be greatly increased.

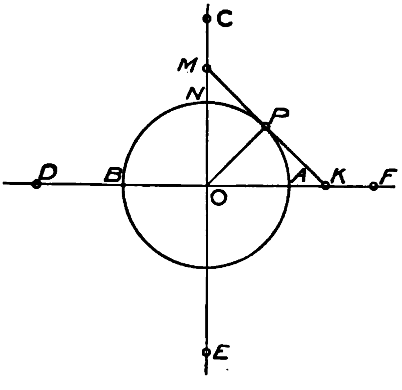


Fig. 39.

Let KP be extended, making PM equal to KP. At the beginning of the motion, KM will coincide with the horizontal center line and as P moves through the first quadrant K will slide to O and M will be pushed upward to C. While P moves through the second quadrant *K* will slide to *D* and *M* will return to *O*. As *P* moves through the third quadrant, *K* returns to *O* and *M* slides down to *E*, and as *P* returns to the origin, *MK* comes back to its origin on the line of centers.

54. Path of Block not on Center Line.—If the direction of the sliding block *K* does not lie through the axis about which *P* rotates, then both the displacement and the acceleration of *K* will be changed. In Fig. 40 suppose that *P* rotates about *O* and *K* slides in a straight line *XX*. From the center *O*, with radius $OP+PK$ cut *XX* at *D*, and from the same center with radius $PK-OP$ cut *XX* at *C*. Then the displacement of *K* is from *D* to *C*, a distance which is less than twice *OP*. Draw the straight lines *DO* and *CN;* then the outward dead point is at *N* and the inward dead point is at *S*.

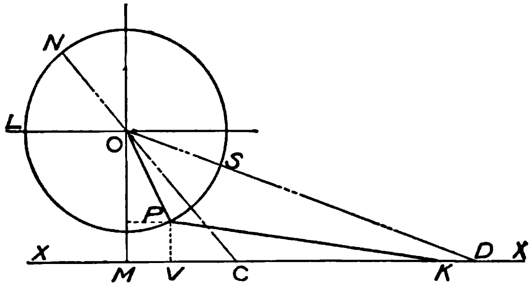


FIG. 40.

Any position of *K* corresponding to a given position of *OP* can be determined thus:

Drop the perpendicular *OM* and let the angle $POM=\theta$.

Then $MK \;=MV+VK$ and $MV=OP\sin\theta$,

$$\text{Also } VK \;=\sqrt{PK^2-PV^2}=\sqrt{PK^2-(OM\pm OP\cos\theta)^2},$$

the positive sign being used when *P* is above the center *O*.

$$\text{From this, } MK=OP\sin\theta+\sqrt{PK^2-(OM\pm OP\cos\theta)^2},$$

$$\text{and } MD=\sqrt{(PK+OP)^2-OM^2}, \text{ where } PK+OP=OD.$$

From these values *DK* and *CK* can be determined.

It may be seen from the figures that *K* moves from *D* to *C* while *P* moves from *S* to *N*, and returns from *C* to *D* while *P* moves over the arc *NLS*. Supposing *P* to move uniformly, the practical result is that *K* moves from *D* to *C*, in less time than it takes to return, the times being proportional to the arcs *SN* and *NLS*.

The velocity and acceleration of *K* at any point in its path may be found by the method of Art. 51.

55. Whitworth's Quick Return Motion.—Cutting tools that must be driven slowly while removing metal may be made to save time by moving rapidly on the return stroke, by making use of the principle just explained.

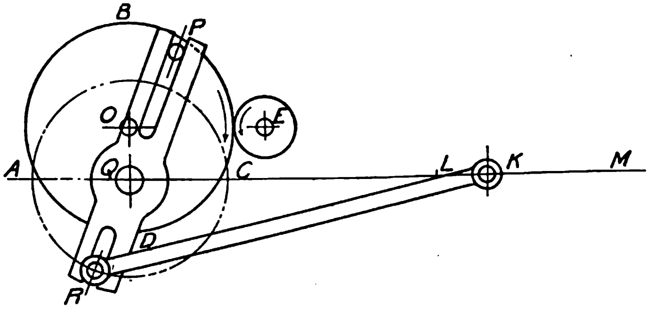


Fig. 41.

In Fig. 41, *O* is the center of a spur gear carrying the fixed pin *P*. *Q* is the fixed center of a rotating arm which is driven by *P*, working in a slot. The other end of the arm has an adjustable pin *R* to which is fastened the link. As the gear revolves, driven

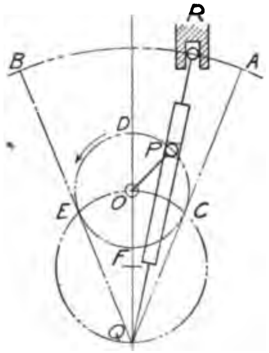


Fig. 42.

by the pinion *E*, the driving arm *QP* varies in length and the angular velocity of *R* varies inversely as *QP*.

Supposing *P* to rotate uniformly in the direction indicated, then *K* will move from *L* to *M* while *P* moves over the arc *ABC*, and return from *M* to *L* while *P* moves over the arc *CDA*. The time of the forward and return strokes will be proportional to these arcs.

A modified form of the Whitworth motion in which the center *Q* lies outside of the crank pin circle is shown in Fig. 42. *P* rotates about the fixed center *O* and drives the slotted arm *QR*, the cutting tool being connected to *R*. If *P* moves uniformly in the direction indicated, *R* will move from *A* to *B* while *P* moves over the arc *CDE*, and return from *B* to *A* while *P* moves over *EFC*, the time of the movements being proportional to these arcs.

56. Pin and Slotted Bar.—In Fig. 43 the crank *OP* rotates uniformly in the direction shown by the arrow and the slotted bar follower slides from *A* to *C* while *P* moves over the arc *ADC*, the motion being harmonic.

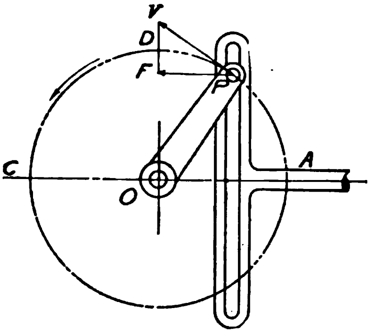


FIG. 43.

Let *PV* be the velocity of the crank at any instant, then *PF*, found by dropping the perpendicular *VF*, is the velocity of the bar at that instant.

$$\frac{PF}{PV} = \sin PVF = \sin POA.$$

Therefore $PF = PV \sin POA$.

The total displacement of the bar is equal to twice the length of the crank, and the length of the slot must equal twice the length of the crank plus the diameter of the pin.

This arrangement is sometimes used for driving pumps and also for light work as in sewing machine drives. In the latter case the slot is generally curved and the arrangement is known as an "inverse cam."

57. Class II. To Transform Continuous Rotation into Rotative Reciprocating Motion.—In Fig. 44, let *OP* be a driving arm and let it make continuous rotations about *O* while the following arm *QK* oscillates over an arc either above or below *Q*. In designing a motion of this class three limitations must be considered:

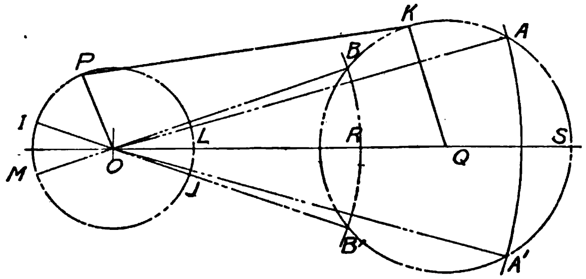


FIG. 44.

(1) The stopping points of the arms.
(2) The dead points of the arms.
(3) The relative length of the arms, link and center distance.

58. Stopping Points and Dead Points.—On *O* as a center, with radius equal to $OP+PK$, intersect the path of *K* at *A* and *A'*. Again from the same center, but with radius equal to $OP-PK$, intersect the path of *K* at *B* and *B'*. *AB* is one of the arcs over which *K* oscillates and *A* and *B* are the stopping points.

If the link and arm be disconnected at *K* and reconnected below the center line, then *K* will oscillate between the stopping points *A'* and *B'*.

Connect *O* and *A* by a straight line. This line will intersect the path of *P* at *L*, and *L* will be the *inner* dead point of *P*. Again connect *B* and *O* by a straight line and produce it to intersect the path of *P* at *M*. Then *M* will be the *outer* dead point of *P*. In the same way the dead points *J* and *I* can be found when the oscillations of *K* are between *A'* and *B'*.

59. Relative Length of Arms, Link and Center Distance.—To transmit the required motion, the follower must have no dead points and the driver no stopping points, also the links of the chain, Fig. 44, must have the following relations:

(a) *QK* must be greater than *OP*.
(b) $OQ+QK$ must be greater than $OP+PK$.
(c) $OQ-QK$ must be less than $PK-OP$.

Case (a).—If *OP* is made equal to *QK* then both arms will rotate about their respective centers, or if the centers be moved far enough apart the arms will simply oscillate. On the other hand if *QK* is less than *OP* the mechanism will be reversed and *OP* will oscillate between stopping points while *QK* will rotate.

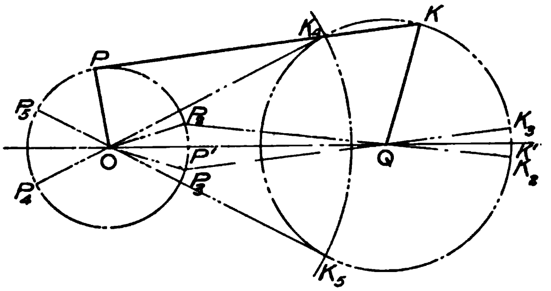


FIG. 45.

Case (b).—If $OQ+QK$ is equal to $OP+PK$ the centers will be moved so near together that both arms will have simultaneous dead points as shown at *P'* and *K'*, Fig. 45. Again if $OQ+QK$ is made less than $OP+PK$, the centers will be moved still nearer together and *QK* will have dead points at K_2 and K_3 from which it is possible for this arm to move in either direction, hence the motion of *QK* will be uncontrolled by that of *OP*.

Case (c).—If $OQ-QK$ and $PK-OP$ are made equal, then there will be simultaneous dead points for the arms at *P'* and *K'*, Fig. 46, and the motion of *QK* will be uncontrolled by that of

OP. Again if $OQ-QK$ is made greater than $PK-OP$ the centers will be pushed so far apart that QK will reach the line of centers at K' ahead of OP, giving stopping points for the driver at P_2 and P_3. Hence OP cannot make complete rotations.

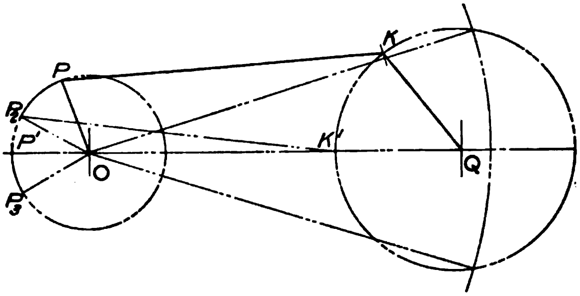


Fig. 46.

60. To Construct a Four-Link Chain of Class II.—If the center distance, the length of the follower arm and the extent of its oscillations are given, the mechanism can be constructed as follows (Fig. 47): Draw the path of K and lay off the arcs AB and $A'B'$, which limit its oscillations. From O as a center draw the arcs AA' and BB', cutting the center line at C and D. Bisect CD at V. Then VC or VD equals the length of the follower arm. For:

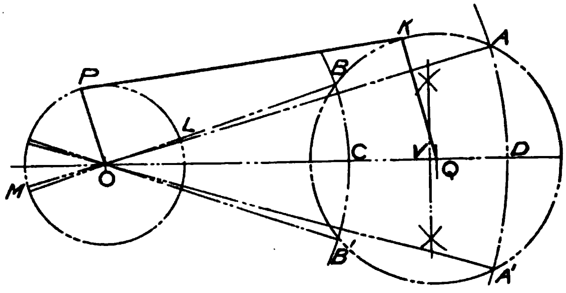


Fig. 47.

CO, or its equal BO, is equal to $(PK-OP)$
and

DO, or its equal AO, is equal to $(PK+OP)$
therefore, $AO-BO=(PK+OP)-(PK-OP)=2OP$
Then CD, which equals $AO-BO$, is equal to $2OP$.
M and L are the dead points of the driver corresponding to the stopping points A and B.

61. Class III. Rectilinear to Rotative Reciprocating Motion.—As the block O in Fig. 48 moves back and forth in the line CD, the arm OP oscillates between P' and P_2, the relative positions and velocities being determined as in the case of the Class I motions.

Either end of the link may be attached to the driving element.

Since O moves in a straight line, it is desirable to have P deviate by as little as possible from that line. This will be accomplished when P is located as follows: Let x be the perpendicular distance from Q to the line CD and let α equal one-half of the angular oscillation of QP, then if QP be taken equal to x plus one-half the versed sine of α, P will deviate from the line CD by an equal amount on either side of that line and consequently its total deviation from CD will be a minimum.

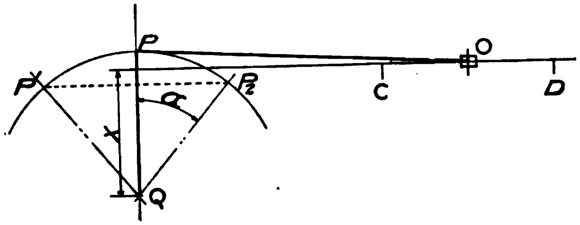


FIG. 48.

The valve rod of a steam engine with its driving arm is frequently made in the form of a linkage of this class, P being the driving end which is connected to the eccentric rod at K, Fig. 38.

62. The Beam.—Fig. 49 shows a mechanism that is used on beam engines to transmit the reciprocating motion of the piston to rotary motion at the crank pin. It is sometimes called a walking beam or working beam. The beam oscillates about the center O and one end receives motion from the piston through the link BB' and the other end transmits this motion to the crank pin through the connecting rod C. BB' is a class III link, in which the amount of oscillation is determined by the length of stroke of the piston. The half length of the beam should be made equal to x plus one-half the versed sine of half the angle of oscillation α.

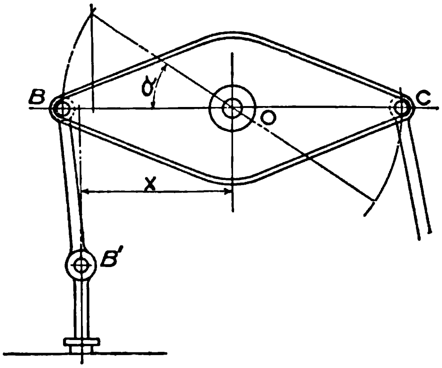


Fig. 49.

Class IV. To Transmit Rotative Reciprocating Motion.

63. The Bell Crank.—This form of linkage is used for changing the direction of motion.

In Fig. 50 let QP be the direction of a reciprocating motion

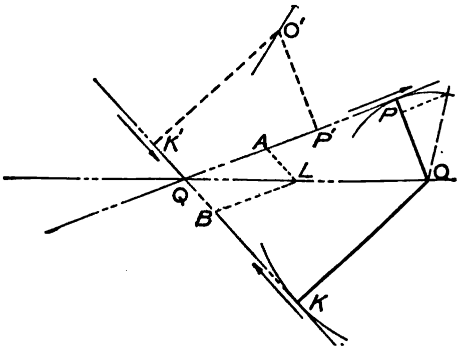


Fig. 50.

which is to be changed to that of QK and let the linear motion of P be equal to one-half that of K.

Lay off from Q on QP, a distance QA, equal to any two units of length, also lay off on *QK* the distance *QB*, equal to one-half of *QA*. Complete the parallelogram *QALB* and draw a line through *Q* and *L*. Locate a point *O* anywhere on this line and draw *OP* and *OK* perpendicular respectively to *QP* and *QK*. Then *O* may be made the center of the bell crank, the arms of which should lie in the direction of the perpendiculars *OP* and *OK*. The length of arm should be equal to the perpendicular plus one-half of the versed sine of the half angle of oscillation.

Since the oscillation is generally small the perpendiculars from *O* to the lines of action are sensibly constant hence the angular velocity is approximately constant. The linear velocity of *O* and *P* will be inversely proportional to the length of the arms.

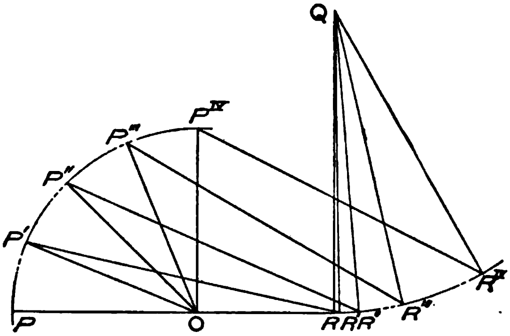


FIG. 51.

The center of a bell crank connecting the motions along *QP* and *QK*, may be placed in any one of the four angles made by these lines at *Q*. With the center at *O*, the relative direction is that shown by the arrows but if the center be taken at *O′*, the direction of the motion at *K* will be reversed.

64. To Produce a Rapidly Varying Velocity from Uniform Motion.—In Fig. 51 let *OP* be an arm oscillating with a uniform velocity about *O*, and connected by a link *PR* to an arm that oscillates about center *Q*. Let the link and arms have such a relative position that when *P* is at a dead point *QR* will be perpendicular to *PR*. Starting from the dead point, the arm *OP* at first produces a small relative motion of *QR*, but as *OP* moves through equal angles *QR* will move through angles rapidly increasing in size. That is, *QR* is rapidly accelerated. When *OP* is reversed in direction *QR* is retarded. This is the principle of the toggle joint, a mechanism that is frequently used on brakes.

65. To Multiply Oscillations by Link Work.—In Fig. 52 the arm *OP* rotates about *O* and is connected by the link *PR* to the arm *QR* which oscillates about *Q*. For one revolution of *OP*, *QR* will move to *QR'* and back again to *QR*. Connect *R* by a link *RL* to an arm *ML* oscillating about *M*. While *OP* turns from *P* to *P'*, *QR* will move from *R* to *R'*, and *ML* will move from *L* to *L'* and back again to *L*. Therefore for every complete revolution of *OP*, *ML* will perform two complete cycles of motion. If *ML* is connected to another oscillating arm *NS*, the latter will complete two cycles for one of *ML* and four cycles for one revolution of *OP*.

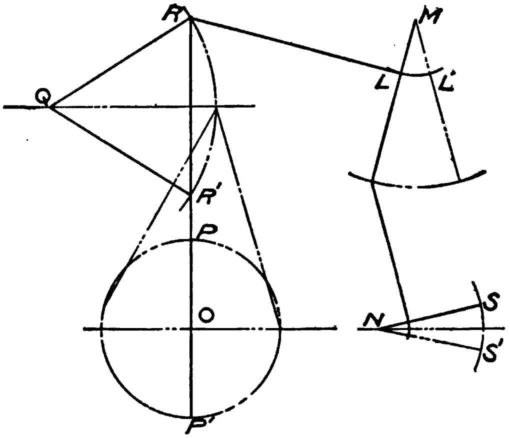


FIG. 52.

66. Class V. Links for Transmitting Continuous Rotation.—In Fig. 53 (*a*) let *OP* equal *QK* and let the link *PK* equal the center distance *OQ*. Either arm can make continuous rotations and can be used to drive the other but the arms will have simultaneous dead points when lying in the direction of the line of centers. Hence the follower arm is uncontrolled by the driver when in this position. To insure continuous rotations of the follower it is necessary to provide some method of getting the link past the dead point. In the case of the locomotive side rod, the cranks on opposite sides of the engine are 90 degrees apart so that when the rod on one side is on the dead points the other is in the position of maximum torque.

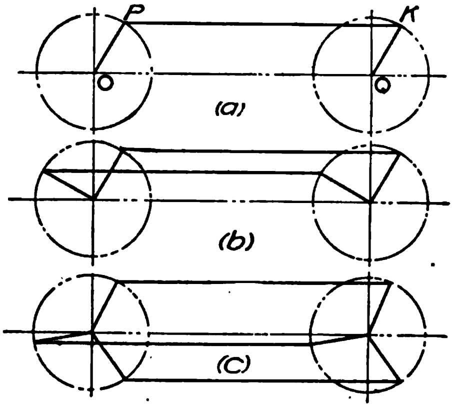


FIG. 53.

Another method of getting past the dead points is shown by Fig. 53 (*b*). Two systems of arms and links are used, the arms being set at 90 degrees apart.

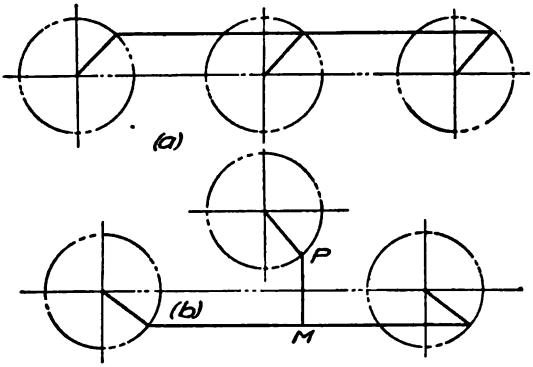


FIG. 54.

Three systems may also be used as at (*c*), where the arms are placed 120 degrees apart.

Fig. 54 shows a method of getting the link past the dead points, in which a third rotating arm is connected to the link. This auxiliary arm must be the same length as the other two and be parallel to them in all positions. The driving arm is placed between the follower and the auxiliary and its center of rotation may be on the same line of centers as the other two as shown at (*a*) or it may be placed to one side as at (*b*). In the latter case the piece *PM*, is rigidly connected to the link.

The velocity ratio of arms of this class will be equal and constant.

67. Drag Link. (Fig. 55).—This arrangement may be used to connect two parallel shafts which are not in the same straight line where a constant velocity ratio is not a prime requisite. In order to give continuous rotations there must be no dead points and consequently no stopping points for either arm. To meet this condition the link, arms and center distance must have the following relations:

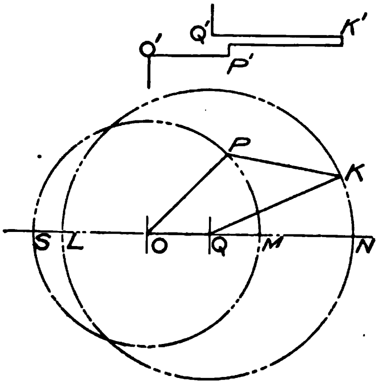


FIG. 55.

$$PK > [(QK + OQ) - OP.] \tag{1}$$
$$PK < [QK + (OP - OQ).] \tag{2}$$
$$OQ < OP \text{ or } QK. \tag{3}$$

From the figure, the value of the parenthesis in (1) is equal to *MN* and that in (2) is equal to *ML*. Therefore the link must be greater than *MN* and less than *ML* if the mechanism is to work properly.

Suppose it is required to design a drag link to connect two shafts a given distance apart.

Locate *O* and *Q* from the given center distance, assume any convenient length for the arms and draw circles to represent the paths of *P* and *K*. Then the link *PK* may be taken any length between the limits given.

To test the mechanism for dead points and stopping points, proceed as follows:

With *O* as a center and radius $OP+PK$ intersect the line of centers on the right. If the intersection falls at *N* then the link and arms will all lie in the same line when *K* reaches *N*, there will be a dead point for this position and the linkage will fail to work. Again if the intersection falls between *N* and the center *O*, there will be stopping points for *K* and the linkage will fail. If, however, the intersection falls beyond *N* on the right, the linkage will work.

Now with *O* as a center and radius $QK-OQ$, intersect the line of centers on the left. The linkage will work only when the intersection falls outside of *L*.

The drag link may be constructed so that the centers *O* and Q coincide and both arms the same length.

The velocity ratio of *P* and *K* is inversely proportional to the perpendiculars from the centers *O* and *Q* upon the line of the link. This ratio is constantly varying.

The upper part of the figure shows the manner of connecting the arms and link so that they will pass each other. $P'K'$ is the link and $O'P'$ and $Q'K'$ are the arms.

68. Boehm's Coupling.—Another Class V motion is shown at Fig. 56. Two disks, *A* and *B*, are placed in parallel planes on the parallel shafts which are to be connected. Two or more links as *L*, *M* and *N* are connected to similar points on the two disks which must be placed far enough apart so that the angle made by the links with the plane of the disks is large enough to allow the links to pass each other in rotating. The velocity ratio of the disks will be constant.

69. Hooke's Coupling or Universal Joint.—This is a link coupling that is used to connect non-parallel axes. The mechanism, as shown in Fig. 57, consists of a pair of semicircular forks placed on the ends of the axes and rotating in planes perpendicu-

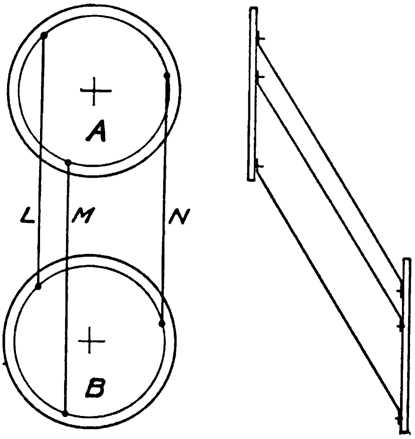


Fig. 56.

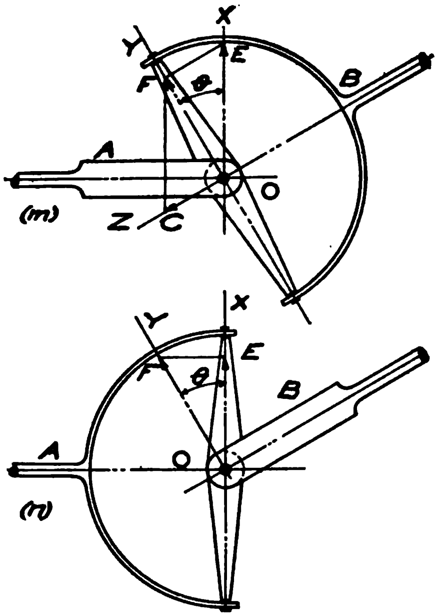


Fig. 57.

lar to the axes. The forks are connected by a rigid cross, having arms of equal length, the ends turning in the ends of the forks.

As the ends of the forks rotate, each in its respective plane, the two arms of the cross which are connected to the driving fork, rotate in a single plane with reference to the driving axis, but the two ends of the cross which are connected to the follower fork rotate so as to have resultants, simultaneously in two planes with reference to the driving axis, one being the plane of rotation of the follower fork and the other the plane of the driver axis.

Suppose the angle between the axes to be θ and their angular velocity ratio to be $\frac{\text{Driver}}{\text{Follower}}=\frac{A}{B}=\frac{\alpha}{\alpha'}$

At (n) the fork A is taken parallel to the plane of the paper and B is at right angles to it. At this instant each of the four ends of the cross lie in the plane of rotation of fork A. The ends which are connected to A move in the plane of OX with velocity OE, while the ends that are connected to B move in the plane of OY. Then OF, which represents the velocity of B, equals $OE \div \cos\theta$.

Hence the angular velocity ratio $\frac{\text{Driver}}{\text{Follower}}=\frac{\alpha}{\alpha'}=\frac{OE}{OF}=\cos\theta$ (1)

At (m) the axes have turned through an angle of 90 degrees so that fork B is parallel to the plane of the paper and A is perpendicular to it. At this instant the ends of the cross connected to A are moving in the plane of OX with velocity OE, at the same instant the ends which are connected to B have two components, one in the plane of OY and the other in the plane of B. The value of either one of these components is the resultant obtained by combining OE with the other component. Then OF, which represents the angular velocity of B, equals $OE\times\cos\theta$.

Hence the angular velocity ratio $\frac{\text{Driver}}{\text{Follower}}=\frac{\alpha}{\alpha'}=\frac{OE}{OF}=\frac{1}{\cos\theta}$ (2)

Expression (1) shows that $\frac{\alpha}{\alpha'}$ is a minimum when the forks are in the position (n) and (2) shows that $\frac{\alpha}{\alpha'}$ is a maximum when they are in the position (m). Therefore if the angular velocity of A is uniform, B will have a maximum angular velocity in the position (*n*) equal to the velocity of *A* divided by the cos of θ. In the position (*m*) it will have a minimum velocity, equal to that of *A* multiplied by the cos of θ.

The velocity of the follower therefore varies from a maximum to a minimum or the reverse every 90 degrees, has two maximum and two minimum values in each revolution and between the maximum and minimum points there are four points where the velocity ratio is unity.

70. The Double Hooke's Joint.—Where constant velocity ratio is not of first importance, the coupling shown in Fig. 58, consisting of two universal joints and an intermediate link, may be used. This is a convenient device for driving the feed motions of drills and milling machines. It is also used on automobile transmissions.

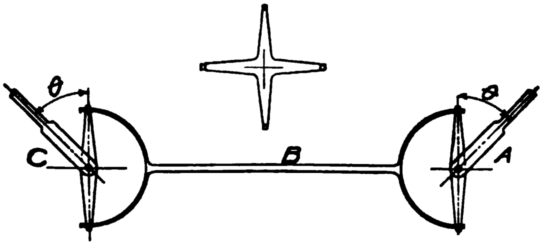


FIG. 58.

If the forks on opposite ends of the link *B* are placed in the same plane, the motion received from *A* will be transmitted again to *C* and if the angle θ is the same at both ends of the link the velocity ratio between *A* and *C* will be constant. If the forks are placed with their planes at right angles, the variation in velocity received at *A* will be doubled at *C*. Hence to give the minimum variation of velocity ratio, with this joint, the forks on the connecting link should lie in the same plane.

71. Pin and Slotted Crank.—Fig. 59 shows a device for transmitting continuous rotation in which the driving arm has constant velocity, while that of the follower is variable.

OP is a driving arm, which rotates about the center *O*. *QR* is the follower arm and it rotates about *Q* as a center. At *P*, in the arm *OP*, is fixed a pin which is made to slide freely in the slot in the arm *QR*.

If the center distance *OQ* is less than the arm *OP*, then *OP*, rotating with uniform velocity, in the direction indicated, will drive *QR*, with variable velocity, in the same direction.

Let *PV*, perpendicular to *OP*, represent the linear velocity of *P* at a particular instant. Draw a line through *P*, perpendicular to *QR*, and draw from this line a perpendicular through *V*, completing the right triangle *PVN*. Then *PN* will represent the linear velocity of the point on *QR* which is in contact with *P* at the instant taken.

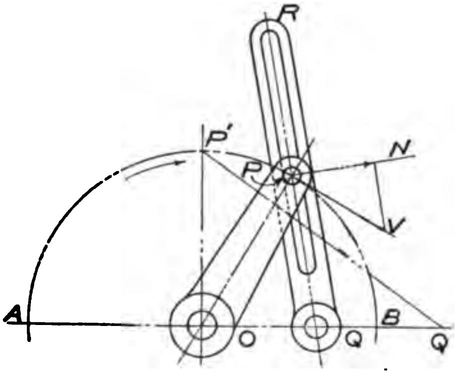


FIG. 59.

Let α and α' be the angular velocity of the driving and following arms at the instant taken. Then

$$\alpha = \frac{PV}{OP} \text{ and } \alpha' = \frac{PN}{QP}.$$

Now $PN = PV \cos VPN = PV \cos OPQ$,
therefore,

$$\alpha' = \frac{PV \cos OPQ}{QP}$$

and

$$\frac{\alpha}{\alpha'} = \frac{QP}{OP \cos OPQ}.$$

The limiting values of the velocity ratio will occur when *OPQ* is zero, that is when the arms lie in the line of centers. When

P is at A, the value of $\frac{\alpha}{\alpha'}$ will be a maximum and when P is at B, its value will be a minimum.

If OQ be made greater than OP so that Q lies outside of the path of P, then OR, instead of making complete rotations, will oscillate across the line of centers through an angle whose sine is $2\frac{OP'}{Q'P'}$.

When Q lies inside the path of P, the length of the slot will be $(AB-BQ)$ plus the diameter of the pin. When Q lies outside the path of P, the length of the slot will be equal to the diameter of the pin plus 2 OP.

72. Aggregate Motions by Link Work.—When a follower link is influenced by more than one driver at the same time its resultant motion is known as a *differential* or an *aggregate motion.* The number of drivers is generally two and never more than three, because each driver determines the direction of at least one point of a follower and the direction of three points in any body completely determines its motion. The principle of aggregate motions may be illustrated by Fig. 60. Let the line AB be given simultaneous motions of rotation and translation such that A is moved to A' with velocity α and B moved to B' with velocity β. Then any intermediate point, as C, will be moved to C'.

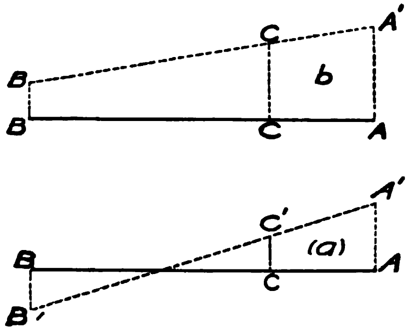


FIG. 60.

Influenced by the motion of A alone with B fixed, the velocity of C will be $\alpha\left[\frac{BC}{AB}\right]$.

If A is fixed and the motion of B only is considered, the velocity of C will be $\beta\left[\frac{AC}{AB}\right]$.

In either figure the velocity of C is equal to the algebraic sum of these two component velocities, or,

$$C=\frac{\alpha BC+\beta AC}{AB}$$

The value of C may be found graphically by erecting perpendiculars at A, B and C and connecting A' and B' by a straight line. The intercept CC' will be the velocity sought.

73. Reversing Link.—The valve gears of reversing engines are sometimes operated by a link which is driven by two eccentrics. The valve rod is connected by a sliding block to an intermediate point on the link so that it is influenced by either eccentric separately. In Fig. 61, O is the center of a shaft carrying the eccentrics B and C. When the shaft rotates, the ends of the link will be moved a distance equal to twice the throw of the eccentrics. The movement of the block F will depend upon its distance from D and E and can be determined by the principle of the previous paragraph. When F is pushed up near D the engine will run in one direction and if moved over to E, the valve will be influenced by the other eccentric and the engine will run in the opposite direction. When F is in the middle position the movement of the valve is too small to uncover the steam ports. The speed of the engine is controlled by adjusting F. This is done in the case of the locomotive, from the cab of the engine.

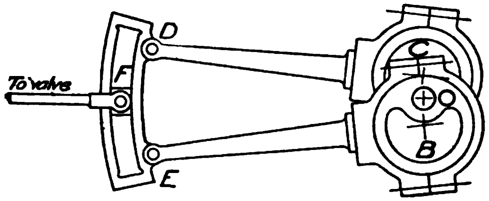


FIG. 61.

74. The Parallelogram.—The term aggregate motion may apply either to the velocity or the path of a body.

One of the simplest devises for producing an aggregate path is the parallelogram, Fig. 62. This is a combination of rigid links fastened together by flexible joints so as to form a rectangle *OACD*. The mechanism oscillates about the fixed point *O*, and any point *P*, on *AD* extended is made to trace the path *PP'*,

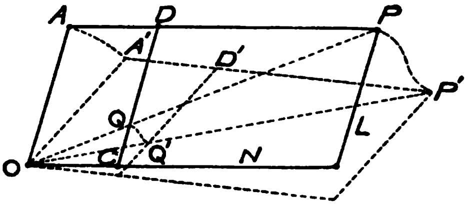


FIG. 62.

then *Q*, on the link *CD* will trace a similar path but reduced in length. In the triangles *POA* and *PQD*, $DQ:OA::PD:PA$, from which $DQ \div PD = OA \div PA$, a constant.

Also $OP:OQ = AP:AD$, a constant.

Then the position of *Q*, on the link *CD*, is fixed and its distance

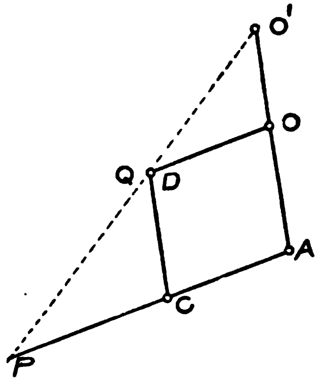


FIG. 63.

from *O* is always relatively the same as that of *P* from the same point, hence the paths of *P* and *Q* will be similar.

The arrangement shown in the figure is one form of the pantograph, a device that is often used for reducing or enlarging drawings. The links *N* and *L* are added to give stability to the tracing point but have no effect on its motion.

The tracing point may be placed either at P or Q according as the drawing is to be reduced or enlarged.

Fig. 63 shows another form of the parallelogram in which the fixed center O is located at O', on the link AO extended. Q is at D and the relative motions of P and Q will be the same as in Fig. 62.

75. Watt's Approximate Straight-line Motion.—An important practical application of the aggregate path is the so-called "straight-line" motion. Fig. 64 illustrates what is generally called the Watt motion. The object of the device is to provide a point of attachment for a follower so that it can be given a rectilinear motion without the use of planed guides.

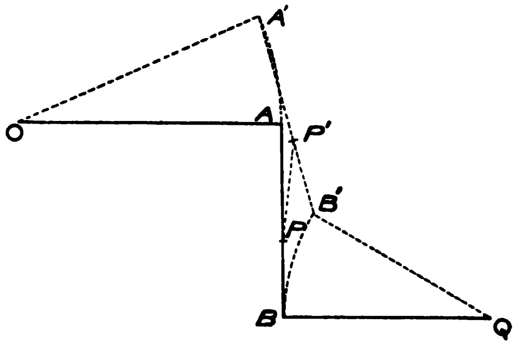


Fig. 64.

OA and QB are two arms turning on the fixed centers O and Q and connected by the rigid link AB. In the position shown by the full lines OA and QB are parallel and AB is perpendicular to both. Now if the arms are moved to a new position such that AOA' does not exceed about 20 degrees, and if P is so taken that $\frac{PA}{PB}=\frac{BQ}{OA}$ then P will move approximately in a straight line.

Fig. 65 shows the Watt motion combined with the parallelogram applied to the beam, piston rod and pump rod of an engine. The links OA, QB and AB make the Watt mechanism and OR, RD, DF and FA are the links of the parallelogram. P is given an approximately straight line motion as in Fig. 64 and D, a point on the parallelogram will have a similar motion but of greater amplitude.

With *O* as the center of oscillation of the beam, *D* is generally connected to the piston rod and *P* to a pump rod. *BQ* is sometimes called a bridle rod.

This mechanism has been much used in the past on long-stroke marine and pumping engines.

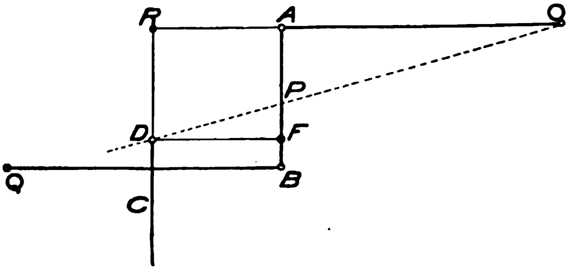


FIG. 65.

76. The Lazy Tongs.—Fig. 66 is a form of the pantograph sometimes used to reduce the motion from the cross head to the indicator on a steam engine.

O is the fixed center of rotation, *P* is attached to the cross head and moves in the line *MN*. Any point *Q*, on the line *OP*, moves parallel to *MN*. Its motion has the same ratio to that of *P* as the distance *OQ* is to the distance *OP*.

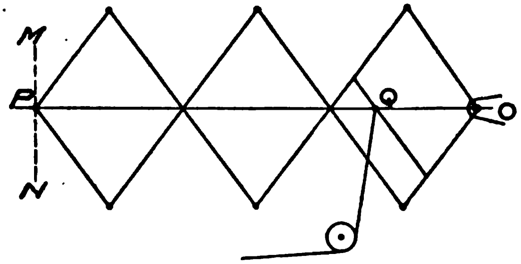


FIG. 66.

77. The Indicator Pencil Motion.—Fig. 67 shows an approximate straight line motion as used on a steam-engine indicator, an apparatus for measuring the steam pressure in the cylinder.

The small cylinder *A* is attached at *B* to a connection leading to the cylinder of the engine. The drum, *C*, is connected, by the chord *F*, to the cross head and is revolved by it. A piston, inside of *A* is exposed to the steam pressure on its lower side and to the action of a spiral spring on its upper side. The alternate action of the steam and spring gives this piston a reciprocating motion, which combined with the rotation of *C*, draws a diagram on a card which is wrapped around the surface of *C*.

The links *OA*, *AP*, *DQ* and *OQ* form a parallelogram in which

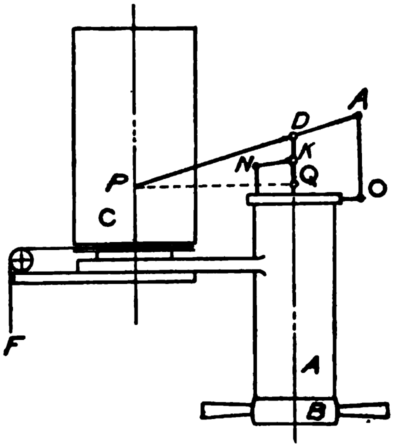


FIG. 67.

Q is the guided point and *P* is the tracing point. *Q* and *P* lie in the same straight line corresponding to *OQP* in Fig. 62.

To give *Q* a straight line motion the short link *NK* is added which with *DK* and *AD* forms a straight line motion similar to the straight-line motion of Fig. 64 except that the guided point is below the points of attachment to the beam and bridle rod instead of between them.

78. Scott Russell Motions.—These motions are shown in Figs. 68 and 69. *C* is a block moving between planed guides and *O* is the center of oscillation of an arm *OR*, which is equal in length to one-half of *CP*. *P* is the guided point moving in the straight line *OP* which is perpendicular to *OC*. This motion is the same as that of Fig. 39 except that *OR* oscillates instead of making complete rotations.

The objection to this as a straight-line motion is that the motion of *C* is determined by planed guides, something which a straight-line motion is supposed to avoid.

The guide may be substituted by a link *QC* as in Fig. 69, called the "grasshopper motion." In this case *P* will have an approximate straight-line motion which may be made to any desired degree of accuracy by increasing the length of *QC*.

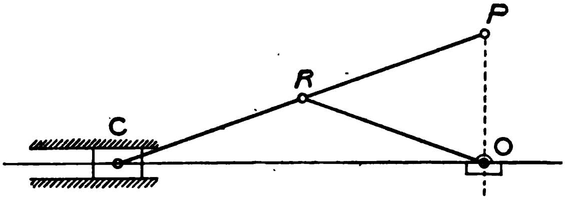


FIG. 68.

79. The Roberts and Russian Motions.—The Roberts approximate straight-line motion is shown in Fig. 70. *OM* and *QN* are equal links which oscillate about the centers *O* and *Q*. *PNM* is a ternary link having its sides *PM* and *PN* equal to *OM* and side *MN* equal to one-half *OQ*. *P* is the tracing point and follows approximately the straight line *OQ* as the links oscillate.

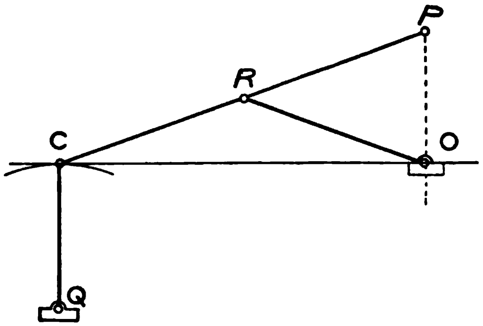


FIG. 69.

The mechanism Fig. 71, known as Tchebicheff's approximate straight-line motion, has the equal arms *ON* and *QM* oscillating about centers *O* and *Q*. The tracing point *P* is located at the middle of *MN* and moves in a line which is approximately parallel to *OQ* when the links have the following proportions: *OM* and $ON = 5$. $MN = 2$. $OQ = 4$.

Both of these motions are very nearly exact.

80. Peaucellier's Exact Straight-line Motion.—Fig. 72, *OB* and *OC* are equal links oscillating about the center *O*. At their extremities are connected four shorter links having flexible joints at *B*, *E*, *C* and *P*. *DE* is a link which oscillates about the fixed point *D*, this point being so located that the distance *DO* equals the length of the link *DE*. *P* is the guided point which is to move in a straight line. From the symmetrical construction of the figure *O*, *E* and *P* always lie in a straight line. Then if the linkage be moved from the central position shown by the dotted lines to some other position, the rhombus *BECP* will be elongated in the direction *EP*, by the link *DE*. The effect is to push the point *P* away from *O* by the same amount that the

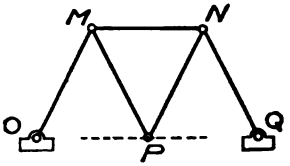


FIG. 70.

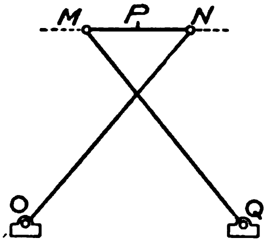


FIG. 71.

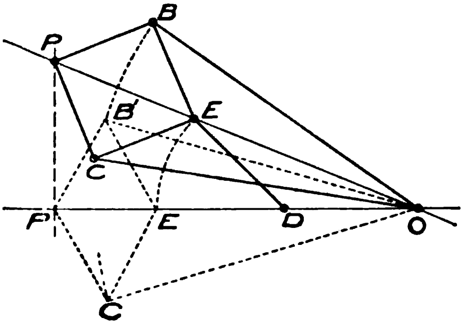


FIG. 72.

increased angularity of the links *OB* and *OC* tend to pull it toward *O*.

PROBLEMS

1. A crank 1 ft. long has a link 3 ft. long. Find the distance of the end of the link from the beginning of stroke when the crank is at 90 degrees from the line of centers. Direction of motion is through center of crank.

2. Find the relative velocity of crank and link in (1) when crank is at 60 degrees from its initial position. (a) By method of components. (b) By methods of instant centers.

3. Draw a velocity diagram for the crank and link of Prob. (1).

4. What is the difference between a cam and an eccentric? Between an eccentric and a crank?

5. An engine crank is 9 in. long and the connecting rod 3 ft. Find the position of the crank when the cross head is in the middle of the stroke.

6. Show how to lay out a quick return motion so that the return stroke will be made in one-third the time of the forward stroke.

7. One arm 3 in. long and another 7, are connected by a link 10 in. long. Find dead points and stopping points for the arms if the centers are 11 in. apart.

8. A link 4 in. long connects arms that are 2 in. and 1 in. long. Find dead points and stopping points for each arm if the center distance is 3 in.

9. Show whether the construction is possible if the center distance in Prob. 8 is reduced to $\frac{3}{4}$ in.

10. Lay out a 90-degree bell crank to transmit a velocity ratio of $\frac{2}{3}$ between two parallel straight lines.

11. In a double universal joint prove that the variation in velocity ratio will be doubled if the forks on the driver and follower are at right angles to each other.

12. Find the length of slot in a bar when the center about which the bar rotates is outside the crank-pin circle. Length of crank 4 in., diameter crank pin $\frac{1}{2}$ in. Distance between centers 5 in.

13. In Fig. 61 if the throw of the eccentrics is 4 in. and the distance between *E* and *D* is 10 in., find the stroke of the block *F* when it is 7 in. from *E*.

14. Prove that the point *P* in Fig. 64 will move in a straight line.

15. Prove that the pantograph will duplicate the outline of an ellipse.

16. Find graphically all the instant centers and points of common velocity in a four-link chain and the velocity of a point in the middle of the movable link.

17. In Fig. 41, the stroke of *K* is 4 in., length *RK* 10 in., ratio of cutting to return stroke 4 to 1, and the distance from *O* to *Q* 2 in. Lay out the mechanism with the ram *K* in the middle of its cutting stroke and find its velocity at that position supposing *P* to make 20 revolutions per minute. Find the velocity of *K* at the middle of the return stroke. Find the time of

the forward and return strokes in minutes. Find the position of *K* for maximum velocity.

18. The driving fork of a double Hooke's joint makes 100 r.p.m. Find the velocity of the intermediate link at the 30-degrees, 45-degree and 60-degree positions.

19. In Fig. 61, suppose the distance from the center *O* to the center of the eccentrics to be 4 in. Find the position of *F* to give the valve rod a movement of 1½ in.

20. Find all the axes and points of common velocity in the Scott-Russell motion. In the Roberts straight-line motion.

CHAPTER V

INTERMITTENT MOTION BY LINKWORK

81. Ratchets.—Ratchets may be classified according to their usage as:

(a) Running ratchets.
(b) Stationary ratchets.

Reuleaux divides ratchets into six classes as follows:

(1) Simple running ratchets.
(2) Continuous running ratchets.
(3) Releasing ratchets.
(4) Escapements.
(5) Checking ratchets.
(6) Locking ratchets.

The first four of these belong to class (a) and the last two to class (b).

Every complete ratchet motion involves three elements, the runner, the click and the pawl or detent. Of these the runner generally receives intermittent motion from the click and the pawl serves to prevent a backward motion of the runner.

Running ratchets are often used for feed motions on automatic machinery and stationary ratchets serve as automatic locks to prevent motion of some part of a machine.

82. The Simple Running Ratchet.—The running ratchet is shown in its simplest form in Fig. 73. Turning about *O* is a ratchet wheel commonly of the shape shown. A lever arm *A*, carrying a click *B*, oscillates about Q and gives an intermittent motion to the wheel. *A* pawl *D* is so placed that it catches the wheel teeth and prevents backward motion.

In action, *A* is pulled backward to the right until the click drops to the bottom of the straight side of the tooth. It is then pushed forward, carrying the wheel tooth with it, through an angle equal to the arc subtended by one or more teeth. When the arm reaches the end of its forward motion and begins to return, the pawl drops into place against the straight side of a tooth and prevents the wheel from running backward.

83. Thrust Between Ratchet Tooth and Pawl.—In a properly designed ratchet:

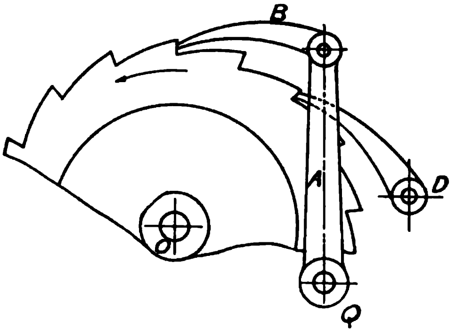


Fig. 73.

(a) The teeth must be of such shape that the click will give to the wheel only forward motion and the pawl prevent only backward motion.

(b) The line of thrust between the pawl and a tooth should pass through the center of rotation of the pawl, and the line of thrust between the click and a tooth should pass through the center about which the click rotates.

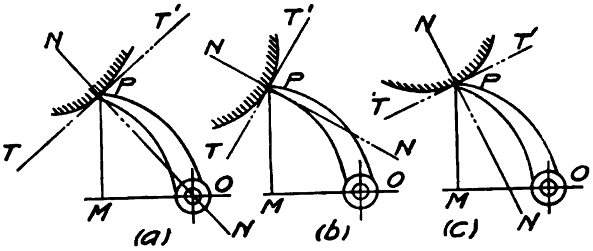


Fig. 74.

In Fig. 74, *TT'* is the line of a straight tooth flank or the tangent to the tooth flank if the latter is curved. *NN'* is called the thrust normal and is perpendicular to *TT'*.

The best position for this normal is shown at (*a*) where it passes through the center *O*.

At (*b*) the normal is shown passing above and to the right of *O*. In this position the ratchet tooth tends to throw the pawl out of contact with it.

At (*c*) the normal falls to the left and below *O*. Here the tooth and pawl have a tendency to jam together.

84. The Silent Ratchet.—The motion transmitted by the ratchet of Fig. 75 is the same as that of Art. 82, without the noise and wear.

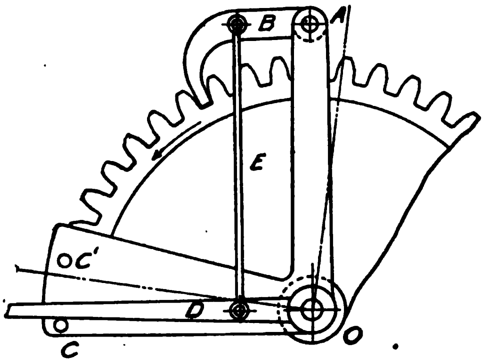


Fig. 75.

The click *B* is carried by one arm *OA* of a bell crank lever which has the same center *O* as the ratchet wheel. The other arm of the lever has two pins *C* and *C'*. Between these pins is the driving arm *D*, also centered at *O* and connected with the click by the link *E*. A downward motion of the arm *D* drives the wheel in a direction shown by the arrow. When the motion of *D* is reversed it first moves back against *C'* before it can move the bell crank lever: During this motion the link *E* lifts the click clear of the teeth. The lever then presses against the pin *C'* and moves the bell crank backward. Then as the driving arm moves down again the click drops in between the next pair of teeth and the wheel is again pushed forward.

A pawl may be similarly designed to hold the wheel from turning backward when the click is raised.

85. Reversible Ratchet.—In the feed motions of shapers and planers, a ratchet similar to that shown by Fig. 76 is sometimes used. The object is to produce a feed motion that can be easily reversed in direction. The engaging side of the click and both sides of the teeth are generally made straight as shown. The reverse side of the click is made of such shape that it can be pulled back over the tops of the teeth without catching on them. The click is centered at B on the arm A, which oscillates about the center O. If the ratchet is driving in the direction shown and it is desired to reverse the mechanism, the click is thrown into the reverse position, shown by the dotted lines.

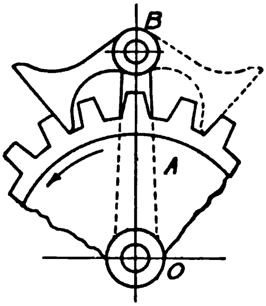


FIG. 76.

With this ratchet it is necessary to have the click travel, on the return stroke, over an arc somewhat greater than that through which it pushed the wheel. This excess should be only just enough to clear the desired number of teeth on the back stroke with the smallest possible amount of lost motion.

86. Double Acting Ratchet.—The forward motion of the wheel may be made practically continuous and the retaining pawls dispensed with by using the double click shown in Fig. 77.

The clicks are mounted on opposite ends of a bell crank which is centered at Q. If the bell crank be rotated in the direction of the arrow, click D will engage a tooth and push the wheel forward while at the same time click N will be drawn backward ready to engage a following tooth.

The arms QH and QK are equal in length and Q is so located that when the bell crank is in its middle position the center lines of the arms are perpendicular to the line of thrust of the respective clicks.

87. Multiple Ratchet.—In the ratchets thus far described, a single forward motion of the click will advance the wheel through an angle equal to the arc subtended by one or more teeth. In Fig. 78 is shown a ratchet designed to advance the wheel through any fraction of the pitch angle. In this case the angle is equal to one-third of the pitch angle.

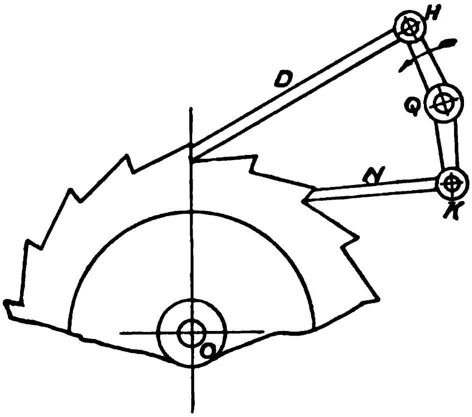


Fig. 77.

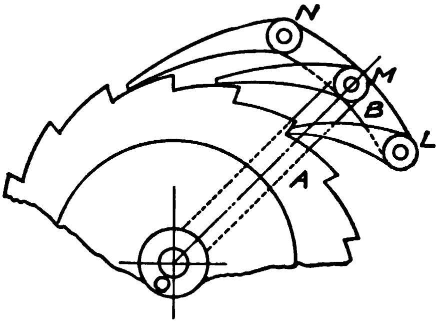


Fig. 78.

The lever *A* oscillates about the same center *O* as that of the wheel. The rocker *B* carries the clicks, which oscillate freely about their centers *L*, *M* and *N*.

In action the arm A is pulled back until click L drops into contact against the side of a tooth. A is then pushed forward carrying all three of the clicks with it and advancing the wheel by an angle equal to one-third of that subtended by a tooth. On the second back stroke click M drops into contact with a tooth and the forward motion is repeated. On the third stroke, click N advances the wheel, the other two remaining inactive. By increasing the number of clicks, the individual movements of the wheel can be made equal to any convenient fraction of the pitch angle.

If N is the number of clicks, they must be so placed that the distance between consecutive contact ends is equal to $\frac{N+1}{N}$ multiplied by the pitch of the wheel.

88. Friction Ratchet.—Intermittent motion may be transmitted by the ratchet shown in Fig. 79 which makes use of neither the pawl nor the click, the wheel being carried forward by the friction between the catch and the rim of the ratchet wheel. This form of ratchet has been considerably used in sawmill feed motions and for operating ship windlasses. In the latter case the backward motion of the wheel is prevented by a second catch which works alternately with the first one. Let RR' be the rim of a wheel and let D represent a saddle which rides on its rim. The lever A is centered at C on the saddle and has its other end connected to a link B. From C is suspended a link and block E. This block just clears the inner surface of the rim when the lever A is in the vertical position.

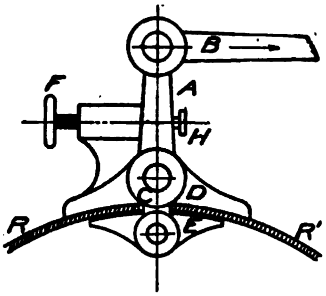


Fig. 79.

If the link *B* is pulled in the direction of the arrow, the lever *A* and the link *CE* fall out of line and the rim is nipped between the saddle *D* and the block *E*. As *A* continues to move to the right the wheel is pulled around by it. When *A* is pushed back, a stop prevents it from turning more than enough to loosen the hold of the saddle and block on the rim.

If it is required to throw the ratchet out of action a second stop *H* may be brought up to the lever by means of the screw *F* so that it cannot turn far enough to the right to allow the block and saddle to nip the wheel rim. No motion will then be given to the wheel.

One advantage of the friction ratchet is that the stroke, need not be an aliquot part of the circumference of the wheel.

89. Releasing Ratchets.—Ratchets of this class are used to hold some part of a machine pending the movement of some other

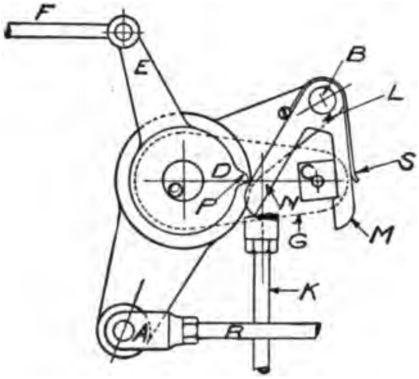


Fig. 80.

part and then to release the part at a given time. A common example is the trip motion of the Corliss valve gear (Fig. 80). *O* is the center of a rotating valve which is actuated by the lever *G*, through the bell crank *AOB*. At *B* is centered a latch link *L*, having two arms, *N* and *M*. One arm has a notch which engages a latch block *C* while the flat spring *S* presses the other arm against the cam *D*, known as the trip collar. The arm *E* of the trip collar receives motion from a link *F* which is attached to the governor of the engine. The latch block *C* is rigidly connected to the lever G and at a point on G, between C and O, a rod K is attached. To the other end of K is fastened the piston of a dash pot.

The operation is as follows: Let the valve be opened by pulling R to the right. This operation raises the end B of the bell crank AOB, carrying with it the latch link L and the latch block C. After being raised a certain distance the end N of the latch link meets the point of the cam at P and is pressed outward by it. This movement pushes the notch on the latch link away from the latch block and releases the lever G. The latter drops by its own weight and the suction of the dash pot and closes the valve.

The time of closing the valve is regulated by the pull of the governor on the rod F. This shifts the trip collar about the center O so that the point P of the cam strikes the latch link at an earlier or later point of its stroke.

90. The Escapement.—This is a mechanism in which one member, under the action of an impelling force, is allowed a regular intermittent motion in one direction.

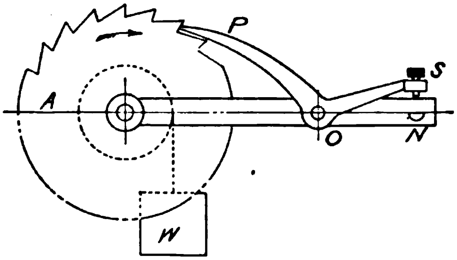


FIG. 81.

The impelled member is some form of toothed wheel and the controlling members are called pallets.

Fig. 81 may be used to illustrate the principle of the escapement. A is the wheel having the drum B around which is wound a cord suspending the weight W. The pallet P is centered at O and so balanced that it will remain in contact with the wheel teeth unless the screw S is held down against the stop N.

The wheel is impelled forward by the weight in the direction shown by the arrow. If S be suddenly forced down and then released *P* will fly out of contact with the wheel tooth, the wheel will turn through one or more pitch distances when the pallet will fall back into place and arrest its motion.

Many modifications of this movement have been invented in the construction of watches and clocks. They are somewhat too specialized to be within the scope of this book but on account of its general application, one movement will be described.

91. The Pendulum Escapement.—In Fig. 82 the balance wheel *A* is acted upon by a spring or a weight which impels it in the direction shown by the arrow. At *B* is suspended a two-arm lever, called an anchor, having pallets *C* and *D* which engage the teeth of the wheel. A pendulum is attached to the anchor in such a way that both oscillate together in a vertical plane.

The swing of the pendulum is represented by the arc *F G H I*. The wheel teeth are made somewhat as shown, with curved sides and oblique ends. The lower side of the pallet *C* and the upper side of *D* are made arcs of circles drawn from the center *B*. The ends of the pallets are finished by oblique lines.

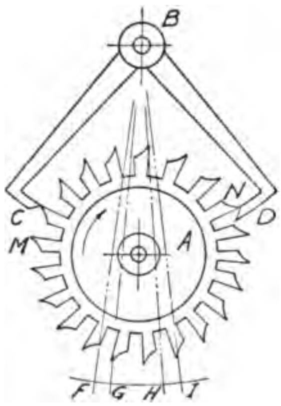


Fig. 82.

Suppose the pendulum to be in the position *G* and swinging toward *F*. In this position a wheel tooth *N* is resting on the upper surface of *D*. The wheel is locked against rotation and the tooth *N* slides on the upper surface of *D* while the pendulum swings from *G* to *F* and back to *G*. The only force resisting this motion is the slight friction between the surfaces of the pallet and the tooth. When the pendulum reaches *G* on its return, the point of a tooth meets the oblique end of *D* and from *G* to *H* presses against this oblique surface thus urging the pendulum forward. When the pendulum reaches *H*, this tooth escapes from the lower point of the pallet and a tooth *M* on the other side of the wheel falls upon the lower surface of *C*. From *H* to *I* and back to *H*, tooth *M* presses against *C* and the wheel is again locked. At *H* the point of tooth *M* meets the oblique end of the pallet, forces the pendulum back from *H* to *G*, during which motion *M* escapes.

The impulse being given to the pendulum at the middle of the oscillation each time, its period is unaffected by the motion of the escape wheel so that the pendulum is made the absolute regulator of the escapement. The escape wheel comes to a dead stop after each escapement.

This pendulum escapement illustrates the principle of mechanical regulators of various sorts which are known as "dead beat." This expression means that the regulator or governor, while dependent upon the movement of the regulated part for being put into operation, is not dependent upon the governed part for stopping its operation.

Thus in a turbine water wheel, the governor is put in operation by an increase in the speed of the wheel above the normal, and the governor, through its connected mechanism, starts to close the gates of the wheel. Now if the governor should keep on closing the gates until the speed of the wheel had fallen enough to automatically stop the action of the governor, the inertia of the mechanism would carry the closing operation too far and the speed of the wheel would be carried below the normal. The governor is therefore made so as to stop its own action independently of the falling speed of the wheel.

92. Checking ratchets are not used to transmit motion, but to check a motion already in progress and bring the moving piece gradually to rest. The general principle is illustrated by Fig. 83 which represents a safety device for an elevator.

AB is a vertical shaft and *C* and *C'* are collars which may be taken to represent the load which is carried by the rod *K* and which moves freely up or down the shaft. *D* is a friction block, attached by means of the link *E* to the center *O*. Q and *O* are both flexible joints. *E* is connected by flexible joints, through the link *F* and the lever *H*, to a rod *K* which is in turn rigidly fastened to the loaded collar *C*. *R* is the center of rotation of *H* and *L* is a flexible joint. A tension spring *S* is fastened to *H*, tending to rotate it in the clockwise direction.

While the load is on *K*, *L* is held up against the action of the spring. *D* is forced downward and held clear of the shaft. If a break occurs, *L* is released, the spring rotates the lever and *D* is pulled up against the shaft. The friction caused by the downward movement of the load jams *D* hard up against the shaft and brings the whole apparatus to rest.

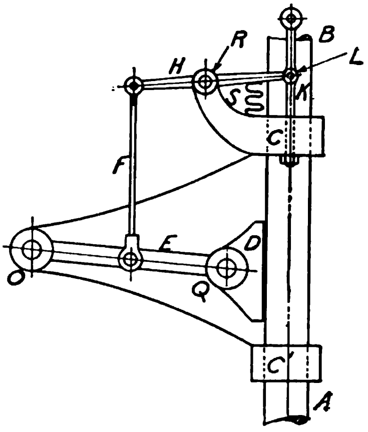


Fig. 83.

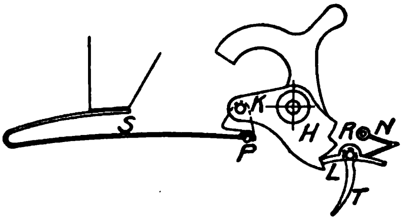


Fig. 84.

93. Locking ratchets are stationary ratchets designed to hold parts of a mechanism in a fixed position against the action of an external force and yet in such manner that they may be readily released.

This form of ratchet is commonly used in fire arms and known as the "lock," an illustration of which is shown in Fig. 84.

The tumbler *H*, to which the hammer is fastened, is the piece to be locked and released. *S* is a spring which pulls downward on a pin *P*, located in one end of a link, the other end of which is connected to the tumbler at *K*. *L* is a trigger lever which acts against *H* and *N* is a spring. The trigger end of *H* is made with notches or teeth like a ratchet wheel.

In the sketch the hammer is shown half cocked and by pulling it back until *L* engages the notch at *R* it may be full cocked. If in that position the trigger *T* be pulled backward, *L* will slip off of its seat against the notch, *H* will be released and its front end will be instantly pulled down by the spring *S*.

The mechanism is generally so arranged that it will not pull off at half cocked.

The spring *N* brings *L* back into position to engage the ratchet, when the hammer is again cocked.

94. The Star Wheel.—Fig. 85 illustrates a form of ratchet, motion, though not strictly a link motion, that is sometimes used on speed counters and similar devices. *S* is a star wheel having three or more teeth. *T* is a rotating lever or tappet which is made to engage the teeth. As the tappet rotates about *O* it strikes a tooth of the wheel once every revolution and turns the latter through one pitch distance. The star wheel is attached to a numbered scale so that every time a tooth is moved a new digit is exposed. The star wheel is thus made to register the number of revolutions of the tappet.

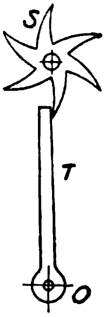


Fig. 85.

PROBLEMS

1. Show the proper direction for the line of pressure between a ratchet tooth and its pawl or detent.

2. Make a sketch of the reversible ratchet on the feed motion of some shaper.

3. Lay out a double-acting ratchet to drive a 20-tooth wheel, the pitch of the wheel being 1 in.

4. If a multiple ratchet has four clicks and the pitch of the wheel is 1 1/2 in., what will be the distance between their contact ends?

5. Lay out a releasing ratchet like Fig. 80 for the following dimensions: Stroke of *R*, 5 in. Maximum lift of *C* 2 in. Average lift of *C* 1 in. Arc of rotation of *E* 15 degrees.

6. Define a "dead beat" escapement and state what use it has.

7. Show how to make the tumbler of a gun lock so that the hammer will not pull off at half cocked.

CHAPTER VI

WRAPPING CONNECTORS

95. Definition and Classification.—A wrapping connector is a flexible cord, belt or rope, which passes around the connected pieces forming a closed loop or endless chain. It may be used when the pieces to be connected are so far apart that gears or links cannot conveniently be employed.

Wrapping connectors may be classified according to their form and the materials from which they are made, as:

(1) Round cords.
(2) Flat belts.
(3) Hemp or wire ropes.

96. Contact of Wrapping Connectors.—The contact of wrapping connectors is theoretically pure rolling since consecutive points on the connector come in contact with consecutive points on both driver and follower. If, however, the force necessary to overcome the inertia of the follower is greater than the force due to the friction between it and the connector, then the latter will slip. Slipping always takes place to some extent when motion is transmitted by wrapping connectors hence the real contact is mixed rolling and sliding.

Since slipping takes place the driving action is not positive.

The directional relation may be either the same or opposite according to whether the connector is "open" or "crossed." In Fig. 86 (*a*) the connector crosses the line of centers between the connected pieces and the pieces rotate in opposite directions as shown by the arrows. At (*b*) the connector does not cross the line of centers and the pieces rotate in the same direction as shown.

If the center line of the connector is taken as the line of action, this line of action will intersect the line of centers in a fixed point A in the case of the open belt, and A' in that of the crossed belt. Therefore according to Rule IV of Art. 41 the velocity ratio should be constant. Owing to the slipping, already noticed, the rule does not hold true and wrapping connectors cannot be used where a constant velocity ratio is essential.

Theoretically the linear velocity of the contact surfaces of the connected pieces and all points of the connector are the same and the angular velocity of the connected pieces is inversely proportional to their radii.

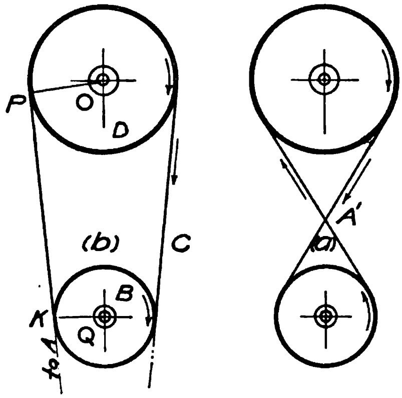


Fig. 86.

97. Cord Connectors.—These may be used for light work such as sewing machine drives, where the power transmitted is small.

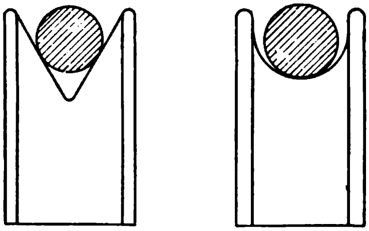


Fig. 87.

Cord connectors are made of cotton, hemp, leather, raw hide or almost any material that is sufficiently strong and flexible.

The pulleys for these connectors are grooved or scored as shown by Fig. 87 so as to keep the cord in its place. The cord rests on the bottom and sides of the groove.

The angular velocity ratio of the connected pieces is inversely proportional to radii, the radius being taken to the center of the cord when the latter is in place. The radii may also be taken to the outside edge or the bottom of the grooves without serious error.

98. Belts.—The flat belt is the form of connector most commonly used for driving machinery. It has the advantages of being light, cheap and flexible and the disadvantage of transmitting an unsteady and variable motion. Belts may be used to connect cylindrical pulleys whose centers are a moderate distance apart, say up to 25 ft. Such belt drives may be run at a speed of 3000 ft. a minute and in some cases they have been run as high as 6000. The best belts are made of strips of leather, cut from the back or toughest part of the hide, and either sewn, cemented or riveted together. For use in damp places where leather belts would disintegrate, belts made of canvas and covered with vulcanized rubber, are suitable. Laced raw hide, waterproof woven cotton and thin strips of sheet metal have been used in some cases.

The common form of a belt drive was shown at Fig. 86 where *D* and *B* are the pulleys and *C* the connector. *D* and *B* may be any size depending upon the power and speed of the connected shafts, they may turn in either direction and either may be the driver. The angular velocity ratio is:

$$\frac{D}{B} = \frac{QK}{OP}$$

The nearer the same size the pulleys can made, the better the belt will work for as the difference in the diameters increases the arc of contact on the smaller pulley decreases and this increases the slipping. When it is necessary to have one pulley much larger than the other their centers must be kept as far apart as possible.

The frictional force between a belt and pulley increases with the width of the belt and the smoothness of the surfaces in contact. A leather belt should have the smoother, or hair side, in contact with the face of the wheel.

99. The Crowned Pulley.—To keep a belt in its place on a cylindrical pulley, the center line of the part which is approaching the pulley should remain in the central plane of the pulley. The constant flapping and vibration of the belt will throw it off sideways unless means are taken to prevent it. This is done by "crowning" the wheel, that is, making its diameter larger on the central plane than at the sides.

The reason why a belt stays on a wheel of this shape may be explained thus:

The pull on the belt and the stiffness of the material tend to keep the edges parallel and running perpendicular to the axes of rotation. When the belt is passed over a cone it tends to lie flat on the face of the cone with edges perpendicular to the elements. These two opposing influences will bend the belt, Fig. 88 (*a*), and in resisting this bending it creeps to the left, to the highest part of the cone and will run off there if not prevented. If another similar cone be placed with its end to the first one, the belt will take a position with its center line over the plane on which the cones are joined and will remain there because the forces which tend to move it to either side of this plane are balanced.

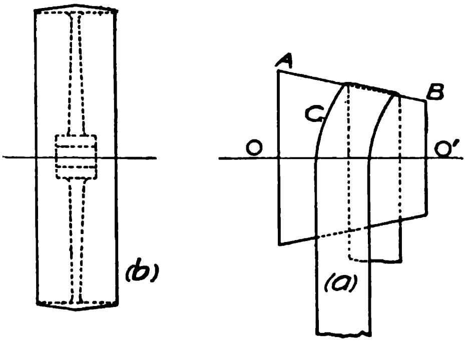


FIG. 88.

A crowned pulley is simply two cones with their large ends placed together as shown in Fig. 88 (*b*).

The amount of crowning that is necessary to hold a belt in place is uncertain; an average value in practice is ½ in. increase in diameter for each foot of increase of width of face. That is, a pulley with a 24 in. width of face would be made ¼ in. larger in diameter on its median plane than at the edges.

100. Shifting Belts.—To put a belt on to a rotating pulley, the part of the belt that is advancing toward the pulley must be started on first. In Fig. 89 a belt advancing as shown must be started at *A* or it will not run on.

Machines are frequently started and stopped without stopping the power that drives them. This is effected by putting both a tight and loose pulley on the follower shaft, Fig. 90. *A* is a wide pulley on the driving shaft and *B* and *C* are two narrow ones on the follower. *B* may be fastened rigidly to the following shaft and turn with it while *C* turns loosely. An arrangement called a "shipper," *E*, presses against the advancing side of the belt *G* causing it to take the position shown by the dotted lines, then the tendency of the belt to keep its edges parallel makes it slide along *A* until it is disconnected from *B* and connected with *C*.

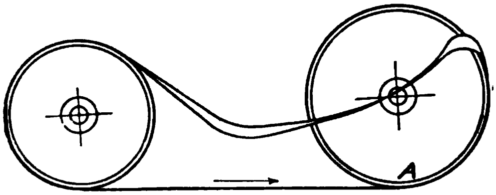


FIG. 89.

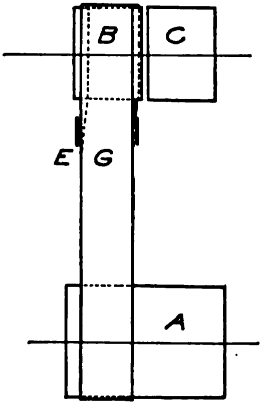


FIG. 90.

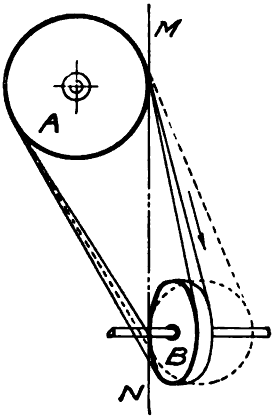


FIG. 91.

101. Twisted Belts.—Two non-parallel shafts may be connected by twisting the belt as shown in Fig. 91.

First assume the shafts to be parallel with the pulleys parallel to the plane of the paper and the belt running in the direction of the arrow. Draw *MN* tangent to each pulley at the center of its face and on the side from which the belt leaves it. Fold over the lower pulley and its shaft on *MN* as a hinge, until the shafts make the required angle. The lower pulley will then take the position shown by the full lines and the central planes of the pulleys will intersect at *MN*. The center line of the belt, as it advances upon the lower pulley, will lie in the central plane of that pulley and the center line of the part that advances upon *A* will lie in the central plane of that pulley. Hence the belt will remain in position as long as the rotation is in the direction shown, but if the direction be reversed it will immediately run off.

102. Guide pulleys are used to change the direction of a belt. In Fig. 92 a belt having the direction *AB* is to be changed to the direction *BC*, the guide pulley must be so located as to be tangent to *AB* and *BC*. Its center will be at some point *O* on *BF* where *BF* is the bisector of the angle *ABC*, made by the two directions of the belt.

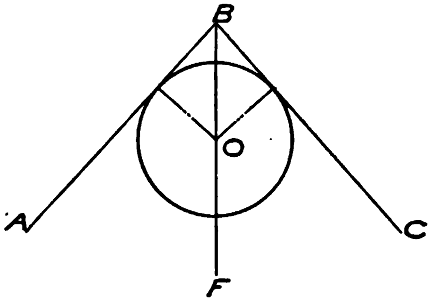


FIG. 92.

If the lines made by the two directions of a running belt intersect so far from the desired position of the guide pulley as to make it of unreasonable size, two guide pulleys may be used instead of one.

Let *AB* and *CD*, Fig. 93, be the two directions assumed for the guided belt. Draw *BC* cutting these lines at convenient points and bisect *ABC* and *BCD*, by the lines *BF* and *CK*. Locate one guide pulley with its center at some point O on BF and tangent to the lines AB and BC. Locate the other guide pulley in a similar way in the angle BCD.

The radius of a guide pulley will be determined by the speed of the belt and should be of such size that its angular velocity will not be excessive.

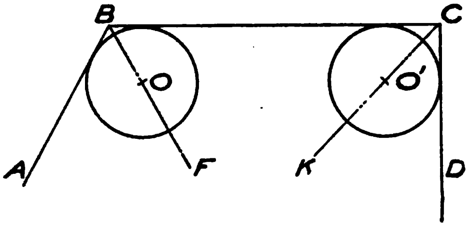


FIG. 93.

103. Guide Pulleys with Non-parallel Axes.—Two shafts whose axes lie in the same plane but which are not parallel may be connected by a flat belt if the pulleys and guides be placed in such relative positions that the element of the surface of one wheel, where the belt leaves it, lies in the central plane of the wheel that follows it. This will cause the center line of the part of the belt that is approaching a pulley to lie in the central plane of that pulley.

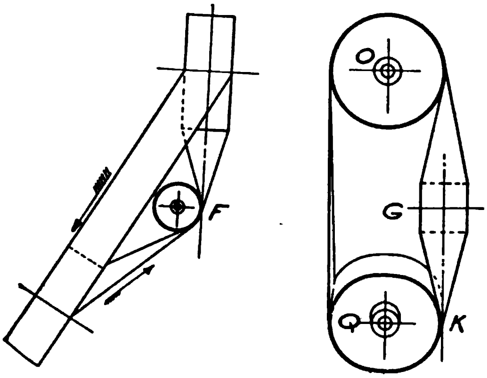


FIG. 94.

In Fig. 94 let O be the driver, Q the follower and G the guide pulley and let the direction of the belt be as shown by the arrows. Place G so that an element at F, where the belt leaves it, will lie in the central plane of O, and at the same time, its central plane contain the element at K where the belt leaves Q. Only one guide pulley is needed in this case, the belt will run only in the direction shown and if reversed it will at once run off.

Shafts that are neither parallel nor intersecting or shafts that are parallel and have pulleys lying in parallel planes may be connected in a similar manner if all of the wheels have the relative positions indicated above.

If guide pulleys be so placed that both the advancing and receding sides of the belt have their center lines in the central planes of the pulleys, any of these systems may be made to run in either direction.

104. Length of Belts. Open Belt.—The length of a flat belt depends upon three factors:

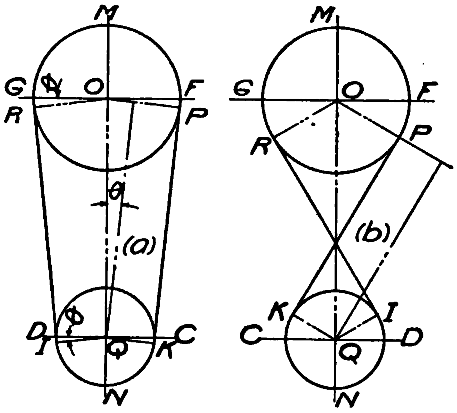


Fig. 95.

(1) The distance between the centers of the connected pulleys.
(1) The diameter of the pulleys.
(3) Whether the belts are open or crossed.

The length is equal to the sum of the two arcs of contact added to twice the distance between the tangent points of belt and pulleys. Thus for the length of the open belt in Fig. 95 (*a*)

$$L = RMP + INK + 2PK \qquad (1)$$

For the purpose of calculation this equation may be expressed in terms of the radii. Draw *FG* and *CD* perpendicular to *OQ* and let θ represent the angle *GOR* or *DQI*, between these diameters and the radii to the points of tangency. Then by geometry:

$$PK=\sqrt{OQ^2-(OP-QK)^2}$$

and from trigonometry, *FP* and *CK* are arcs whose sines are equal to $\frac{OP-QK}{OQ}$. Let the value of these arcs for unit radius be equal to x.

Then since *GMF* and *DNC* are equal to π radians, we have for the arc of contact of the large pulley:

$$OP(\pi+2x).=RMP$$

and for the small pulley:

$$QK(\pi-2x).=INK$$

Substituting values in (1) we have for the total length of the belt,

$$L=2\sqrt{OQ^2-(OP-QK)^2}+\pi(OP+QK)+2(OP-QK)x \qquad (2)$$

In most cases the length may be approximated with sufficient accuracy by making

$$L=\pi(OP+QK)+2OQ \qquad (3)$$

105. Length of Belts. Crossed Belt.—Let the crossed belt be represented by Fig. 95 (*b*) where the symbols denote the same as in the previous article.

In this case, $PK=\sqrt{OQ^2-(OP+QK)^2}$

The arc of contact for the large pulley is,

$$OP(\pi+2x)$$

and for the small pulley,

$$QK(\pi+2x)$$

Then the total length of the crossed belt is,

$$L=2\sqrt{OQ^2-(OP+QK)^2}+(OP+QK)(\pi+2x) \qquad (4)$$

The approximate formula given by Stahl & Woods for this case is

$$L=3\tfrac{3}{8}(OP+QK)+2\,OQ \qquad (5)$$

The safest way to find the length of any belt is by actual measurement over the pulleys. When this is not practicable the belt drive can be laid out to some scale and the length determined by measurement from the drawing. This method should always be adopted as a check on the calculations even when the belt length has been computed as above.

106. Cone Pulleys.—Many machines, like the lathe for example, are required to have different speeds to accommodate the different classes of work. The speed of the driving shaft being constant, the variations may be accomplished with belt drives by using two stepped cones, one on the machine and one on the countershaft from which the machine is driven. Fig. 96 shows the arrangement. The cone is virtually a series of pulleys of different diameters placed side by side. They may have any number of steps, but generally they have from three to six.

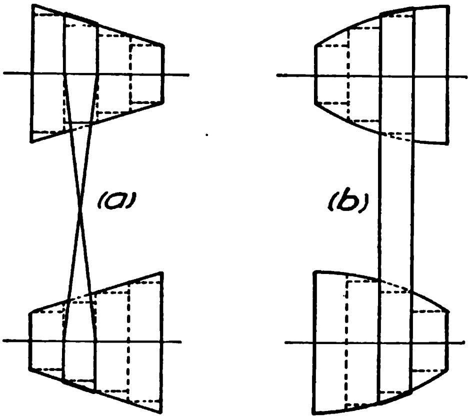


FIG. 96.

As shown by formula (4), Art. 105, the length of a crossed belt depends upon the sum of the diameters of the pulleys, hence a crossed belt will run properly on any two steps of a pair of cones, provided the sum of their diameters be kept constant. A crossed belt is undesirable for use on cone pulleys, on account of the friction of rubbing where the belt crosses, and is therefore seldom used.

107. Stepped Cone for Open Belt.—An open belt will not work properly with a pair of cones like Fig. 96 (*a*). As seen by the formula (2), Art. 104, the length of an open belt depends both upon the sum and the difference of the diameters of the pulleys and both of these factors cannot be kept constant on a pair of cones.

The difficulty may be avoided by making the contour of the cones as shown in Fig. 96 (*b*) and then bringing the corners of the steps out to meet this convex line.

The method of determining the amount of swell, or what amounts to the same thing, the variation in the diameter of the steps, is given by A. C. Smith in the Transactions of the American Society of Mechanical Engineers, Vol. X. This method is satisfactory where the angle made by the belt with the line of centers does not exceed 18 degrees.

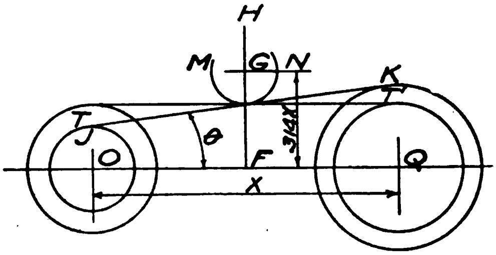


Fig. 97.

In Fig. 97 lay out *OQ* equal to the distance between the centers of the cones. On *O* and *Q* as centers, draw circles to represent the first pair of steps, which must be previously determined from known conditions. Draw the tangent *TT'*. Bisect *OQ* at *F* and erect a perpendicular *FH*. Lay off on this perpendicular a distance *FG* equal to 0.314 *OQ*. With *G* as a center draw the arc *MN* tangent to *TT'*. Now if any line, as *JK*, be drawn tangent to the arc *MN*, circles drawn tangent to it, on the centers *O* and *Q*, will give the circumferences of the cone steps. Hence by assuming the diameter of one step, drawing a line tangent to it and to *MN*, the diameter of the other step can be found. The decimal 0.314 is an empirical value given by the author above mentioned.

108. Rope Connectors.—When the center distance is too great for belts, motion may be transmitted by ropes running on specially designed pulleys.

Rope connectors may be classified as:

(1) Fibrous ropes.
(2) Wire ropes.

Their flexibility, lightness and cheapness gives to rope connectors an advantage over other forms of belting. They may, in fact, be used to transmit motion to distances as great as 500 yds. where the cost and mechanical difficulties of using leather belting would make it prohibitive.

The unsteadiness of the transmitted motion, loss due to friction and trouble of keeping ropes in repair, are among their chief disadvantages.

109. Fiber Ropes.—These are made principally of hemp and cotton. They are conveniently used on long and crooked transmissions inside of factories where their flexibility makes it possible to carry them around corners or from one floor to another in distributing power to various parts of an establishment.

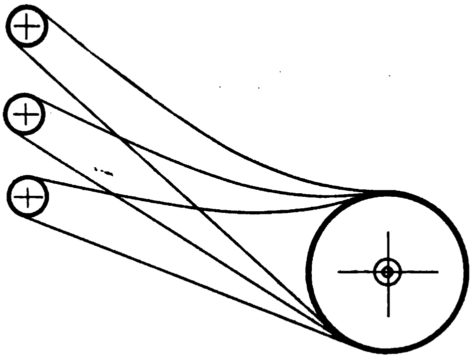


Fig. 98.

There are two systems of fiber rope transmission, known as:

(1) The English system, and
(2) The American system.

These are illustrated by Figs. 98 and 99 respectively. In the English system there are a number of separate ropes, each one being a closed loop and running in a separate groove in the pulleys. The rope is first passed over the pulleys and then the joint made by splicing. This form of drive is suitable for heavy work as between a prime mover and a main shaft. In designing a transmission of this class one or more ropes than are needed should be allowed so that in case one of them breaks it will not cause a shut down of the entire plant.

In the American system one rope runs continuously from one pulley to the next and finally back to the starting-point. Tightener pulleys are used on various parts of the system to keep the rope tension uniform. A disadvantage of this form of rope drive which does not apply to the English method is that a break in the rope will cause a stoppage of the whole system.

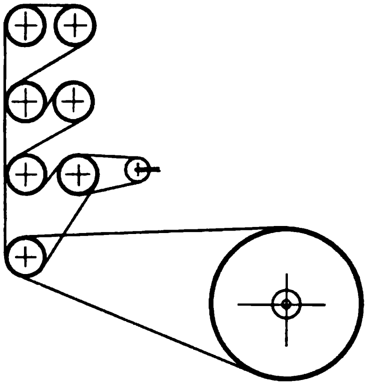


FIG. 99.

110. Pulleys for Fiber Ropes.—These are sometimes made with V grooves as shown in Fig. 100 (*a*). When the work is light the rope may rest on the bottom of the groove, but generally it wedges against the sides so that the frictional resistance to slipping is great. The angle between the sides of the groove should be about 45 degrees. If made much less than this the rope will be squeezed so tightly that it will wear out rapidly. The groove shown at Fig. 100 (*b*) is a more suitable form for heavy work than the V groove. The slope of the sides is the same as before, but they are curved at the top and bottom so that as the rope stretches and wears it sinks deeper into the groove and is not pinched so hard. Where tightener pulleys are used the rope may rest on the bottom of the groove. When the pulley has a number of grooves as in Fig. 98, they must be made the same size and all the same diameter or the rope tension will be unequal and all of the load will be carried by the tightest rope. Considerable loss is due to the bending and unbending of the rope around the pulley. To reduce this as low as possible the diameter of the pulley should be not less than thirty times the diameter of the rope.

The horizontal, unsupported distance between the centers of the pulleys may be from twenty to sixty feet and it is best to have the driving, or tight side of the rope on the lower side of the pulleys. The upper side will then sag down in a curve approximating the catenary, thereby increasing the arcs of contact.

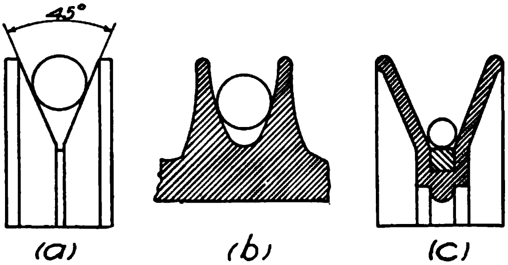


FIG. 100.

The pressure between the rope and pulley is chiefly due to the weight of the rope and the initial tension should not be great enough to pull it tight. The slacker the rope can be, and still do the work, the longer it will wear.

Ropes fitted as above described may be run at speeds all the way from 1000 to 7000 ft. per minute.

111. Wire Rope Drives.—Wire ropes are commonly used for very long drives. In designing a drive of this class it is important to consider:

(a) The form of the groove.
(b) Relative size of pulley and rope.
(c) Vibrations of the rope.
(d) Sag of the rope.

Fig. 100 (*c*) shows the form of groove that should be used with a wire rope. The latter is not supposed to touch the sides at all but lies on the bottom of the groove. Where the pulleys are made of iron the tread of the rope should be of wood, gutta-percha or preferably leather so as to increase the frictional resistance to slipping. The depth of the groove is made large to prevent the rope from jumping out.

If the pulleys can be made of large diameter it will diminish the influence of the stiffness of the rope and decrease the bending force required. The diameter of the pulley should not be less than 100 times the diameter of the rope.

The swaying and flapping of a rope may be great enough to cause serious trouble. This is due to the pulleys being unbalanced or out of line, to the wind or to the bad condition of the rope. If it continues after these defects have been remedied, it may be necessary to use more supporting pulleys.

So-called harmonic vibrations which sometimes occur in short drives may be remedied by changing the speed or by using a supporting pulley.

The driving side of a wire rope is best placed below the pulleys with the slack on top. The rope sags in the form of a catenary but for purposes of measurement the curve may be more conveniently laid out as a parabola.

Roebling recommends that the initial tension be such that the sag of the rope shall be one-thirty-sixth of the span when not running. When the rope is moving the tension on the tight side will be increased so that the sag on that side will be about one-half of that on the slack side. The amount of slack may increase in hot weather owing to expansion.

On long drives supporting pulleys should be placed about every 400 ft. Those on the driving side should be the same diameter as the principal pulleys, those on the slack side may be made smaller since, having a smaller load, they may run at a higher speed.

For very long drives the principal pulleys are sometimes arranged in relays, the first follower shaft having two pulleys, one of which is a follower to the first driver, and the other a driver to the second follower, but such rope drives will not work well where the center distance of the driver and follower is less than 130 ft.

Wire ropes may be run at speeds of from 3000 to 6000 ft. per minute.

112. Wrapping Connectors for Variable Velocity Ratio.—Fig. 101 shows a device that is sometimes used to change the velocity ratio of a driver and follower when the use of stepped cones is not convenient. *W* is a V-shaped belt made by fastening short blocks of wood to a flexible band. At *A* is a pair of disks mounted on a shaft in such a way that they can be pushed together or apart, forming a V-shaped groove of varying width. At *B* is a similar pair of disks mounted in the same way. *L* and *L′* are levers, centered at *Q* and *Q′* and operated by the screw *S*, on which is a chain wheel *K*.

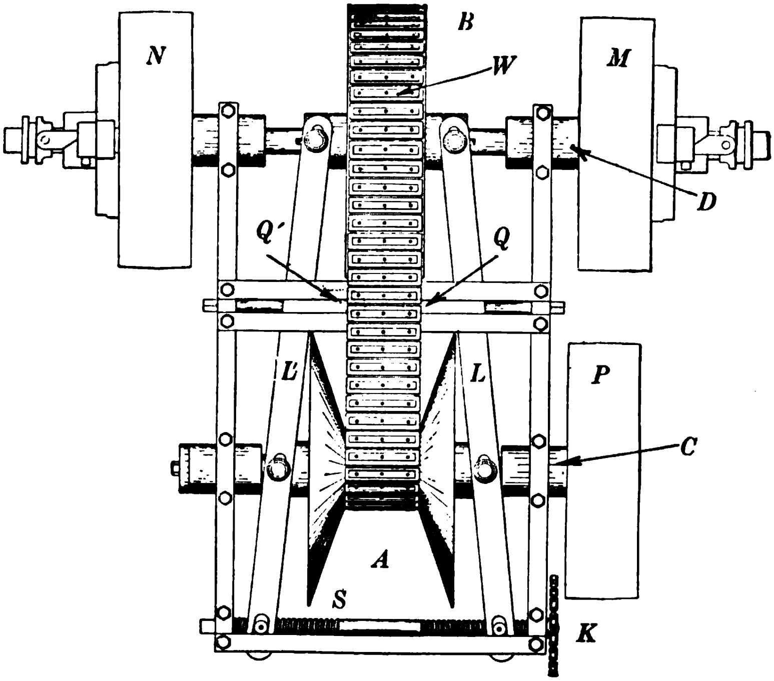


Fig. 101.

Suppose *C* to be the driving shaft connected with a source of motion through the pulley *P*, and let *D* be the follower shaft, which is connected to the driven machine through *M* or *N*:

The operator, by turning the chain wheel, rotates the screw *S* which forces the front end of the levers apart carries the conical disks with them. At the same time the back ends of the levers and the back cones are forced together. The belt drops down between the front cones and is pushed up on the back ones. The effective diameter of the front cones is decreased and that of the back ones increased thus decreasing the velocity of the follower.

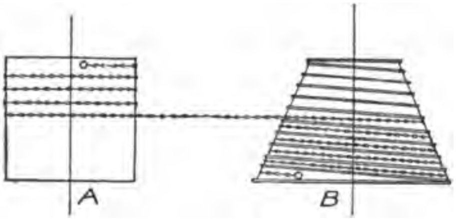


FIG. 102.

113. The Fusee.—Fig. 102 shows a form of wrapping connector that will transmit a variable velocity ratio for a limited period depending upon the length of the connector. The principle is used in some chronometers to counteract the tendency of the time piece to run slow as the spring "runs down."

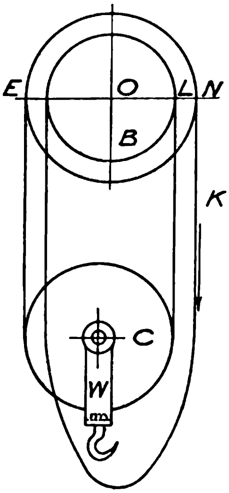


FIG. 103.

The follower *A* is a cylindrical drum while the driver *B* is a conical drum. The wrapping connector is in this case a fine chain having the ends fastened to the drums and arranged to wind and unwind upon them. Inside of the drum *B* is a coiled spring which drives the mechanism. As the spring uncoils and consequently weakens, the velocity of *B* tends to decrease, but the chain is so wound that the effective driving radius at the same time increases. These two tendencies balance each other and *A* is driven at a constant velocity.

114. The Differential Pulley.—Fig. 103 shows a form of wrapping connector which is used for lifting heavy weights. In this case the velocity with which the load is moved depends upon the difference in linear velocity of the circumference of two pulleys of unequal diameter, but which are fixed together on the same axis.

The pulley *B* has grooves of different radii, *OL* and *ON*. An endless chain *K* runs over these grooves and supports the movable pulley *C*, to which a weight may be attached.

By pulling downward on the chain at *K*, the pulley *C* is drawn upward toward *O*.

Let α equal the linear velocity of points *E* and *N* on the larger groove and of a point *K* on the chain. Let α' equal the velocity of points on the smaller groove and β the upward velocity of pulley *C*.

If we assume the chain to be fixed to the pulley at the point *L*, then a downward pull on the chain at *K* will cause the pulley *C* to move upward and its velocity will be $\frac{\alpha}{2}$. Again if the chain be assumed fixed to the pulley at *E*, a downward pull on the chain at *K* will cause *C* to move downward with a velocity $\frac{\alpha'}{2}$. The actual velocity of *C* is the difference of these two velocities, or

$$\frac{\alpha}{2}-\frac{\alpha'}{2}=\beta \qquad (1)$$

Now points at *L* and *N* are both moving downward and their relative velocity is $\frac{\alpha'}{\alpha}=\frac{OL}{ON}$ from which $\alpha'=\alpha\,\frac{OL}{ON}$ (2)

Substituting this value of α' in (1) we have

$$\beta=\frac{\alpha}{2}-\alpha\,\frac{OL}{2ON}=\frac{\alpha}{2}\left(1-\frac{OL}{ON}\right)=\alpha\left(\frac{ON-OL}{2ON}\right) \qquad (3)$$

From which the velocity ratio,

$$\frac{\alpha}{\beta}=\frac{2ON}{ON-OL} \qquad (4)$$

115. Intermittent Wrapping Connectors. Tackle and Falls.—An intermittent motion may be transmitted by wrapping connectors of hemp or wire rope arranged to run over grooved sheaves, one end being attached to a load and the other arranged to wind on a cylindrical drum.

A common illustration is the "Tackle and Fall" shown in Fig, 104, an apparatus which is used for hoisting. The rope, sheaves. blocks and hooks together constitute the tackle and the free end of the rope *P* is called the fall.

The figure shows two sets of blocks and sheaves. From the fall P the rope passes over the outer and larger sheaves and returns around the smaller ones, being finally fastened to the hook K. The whole tackle, including the load, is suspended at C. The fall may be operated by hand or it may be wound over a cylindrical drum.

If P be moved in the direction of the arrow with a velocity V, the load F, will be moved upward with a velocity equal to one-fourth of V. If two sheaves be used instead of four, then the upward velocity of F will be one-half that of V. It follows then that the velocity of the load will be equal to the velocity of the fall divided by the number of turns of the rope or by the number of sheaves.

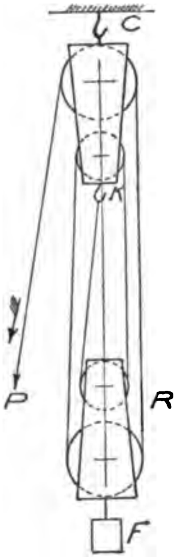


FIG. 104.

This statement must not be applied to the ordinary form of multiple block tackle in which several sheaves are placed side by side, in which case the multiple rope is for strength only and does not affect the kinematical action.

When the fall is wound upon a drum, unless the latter is long enough so that the whole travel of the rope may be wound in a single layer, the travel of the load will be variable as the layers increase on the drum. This is sometimes remedied by placing a second drum behind the first which turns just fast enough to take up the slack rope.

The drums are sometimes made with high flanges at the sides to prevent the rope from running off. They are also made with grooves so that the rope will wind evenly without crossing itself.

PROBLEMS

1. Show why the driving action of wrapping connectors is not positive.

2. Show why the velocity ratio is not constant.

3. Prove that a flat belt will stay on the pulley better when the latter is crowned.

4. Two pulleys are connected by flat belts. The plane of the pulleys makes an angle of 15 degrees and the direction of the belt changes 60 degrees. Locate the guide pulleys.

5. Two shafts are connected by an open belt. Distance between axes 8 ft. Diameter of pulleys 12 and 24 in. Find the length of belt.

6. Find the diameter of steps on a 4-step cone, first step being 12 in. and the last 5 in. in diameter. Belt open. Center distance 10 ft.

7. Design a rope drive 600 ft. long for a 1-in. wire rope. Show the amount of sag. How many supporting pulleys should be used?

8. What are the advantages and disadvantages of the English and American systems of rope driving?

9. A pair of pulleys have diameters of 15 in. and 45 in. and the distance between their centers is 10 ft. Find the length of a flat belt when the latter is open. Crossed.

10. In the differential pulley, Fig. 103, the radii of the grooves *OL* and *ON* are 8 and 9 in. If *K* is pulled down at the rate of 5 ft. per second, how fast will the weight attached to *C* be raised?

CHAPTER VII

TRAINS OF MECHANISM

116. Definition and Classification.—A train of mechanism is a series of pairs of mechanical elements arranged to transmit motion. It includes a driver and a follower and all intermediate connectors, and may be composed of toothed gears, links and levers or belts and pulleys or any combination of these.

Trains are used for three purposes:

(1) To produce a change in velocity ratio.
(2) To change the direction of rotation.
(3) To provide intermediate connectors when a driver and follower are far apart.

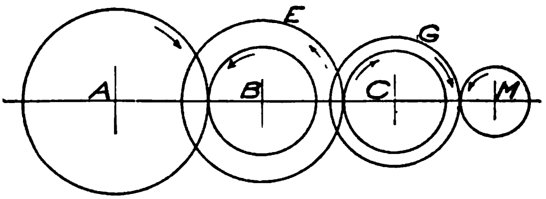


FIG. 105.

Any or all of these features may be combined in the same train.

Kinematically trains may be divided into three classes:

(1) Simple.
(2) Differential.
(3) Epicyclic.

117. Velocity Ratio of a Train.—In Fig. 105 let α_1, α_2, ... α_m, equal the angular velocities of the axes A, B, ... M. Let N_1, N_2 . . . . N_m be the revolutions made by the successive axes in a unit of time, R_1, R_2, ... R_m the radii in inches of the wheels in contact while D_1, D_2 . . . . . D_m represent their diameters and T_1, T_2 . . . . . T_m the number of their teeth if the wheels are gears.

The velocity ratio of this train is the quotient obtained by dividing the angular velocity of the last axis by that of the first axis or the reciprocal of this, that is $\frac{\alpha_m}{\alpha_1}$ or $\frac{\alpha_1}{\alpha_m}$.

But
$$\frac{\alpha_m}{\alpha_1}=\frac{\alpha_2}{\alpha_1}\times\frac{\alpha_3}{\alpha_1}\times\ldots\ldots\frac{\alpha_m}{\alpha_{(m-1)}},$$
also
$$\begin{aligned}\frac{\alpha_m}{\alpha_1}&=\frac{N_2}{N_1}\times\frac{N_3}{N_2}\times\ldots\ldots\frac{N_m}{N_{(m-1)}},\\ &=\frac{R_1}{R_2}\times\frac{R_2}{R_3}\times\ldots\ldots\frac{R_{(m-1)}}{R_m},\\ &=\frac{D_1}{D_2}\times\frac{D_2}{D_3}\times\ldots\ldots\frac{D_{(m-1)}}{D_m},\\ &=\frac{T_1}{T_2}\times\frac{T_2}{T_3}\times\ldots\ldots\frac{T_{(m-1)}}{T_m}.\end{aligned}$$

From this it follows that the angular velocity ratio of the train is equal to the continued product of the angular velocity ratios of the successive pairs of axes, or it is equal to the continued product of the number of revolutions of all the followers divided by the continued product of the revolutions of the drivers, or again it is equal to the continued product of the diameters in inches, radii or number of teeth of all the drivers divided by those of all the followers.

This velocity ratio may also be written:

$$\frac{\alpha_m}{\alpha_1}=\frac{\alpha_2}{\alpha_1}\times\frac{N_3}{N_2}\times\frac{R_3}{R_4}\times\frac{D_4}{D_5}\times\ldots\ldots\frac{T_{(m-1)}}{T_m}$$

Hence it follows that if any of the above properties of a pair of intermediate drivers and followers are known, these ratios between different properties may be combined into a single equation to obtain the value of the train.

118. Directional Relation.—Fig. 105 represents a train of six wheels on four parallel axes all in external contact as shown. If the rotation of the first axis is clockwise, that of the second will be counter-clockwise, that of the third will be clockwise and the fourth will be counter-clockwise like the second. Hence it follows that if the number of axes in a train is odd, the last axis will rotate in the same direction as the first, but if the number of axes is even the last axis will rotate in the opposite sense to the first.

In Fig. 106 there are three axes and three wheels, hence A and C rotate in the same direction. The velocity ratio of this train is

$$\frac{A}{B} \times \frac{B}{C} = \frac{A}{C}.$$

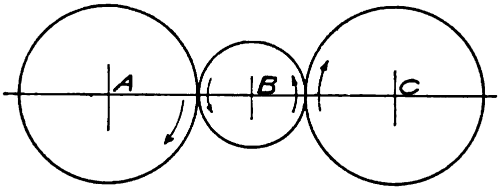


Fig. 106.

In this case B does not affect the velocity ratio, hence it is called an "idler."

In Fig. 107 there are three axes and four wheels. A, C and D are external wheels and B is an internal wheel. As shown by the arrows, if the rotation of A is clockwise that of D is counterclockwise. Hence, if the number of axes is odd and one of them

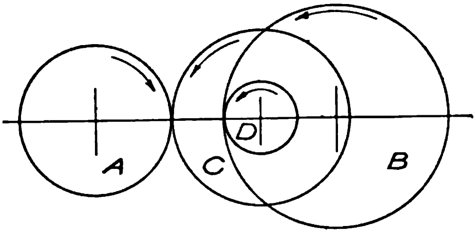


Fig. 107.

carries an internal gear, the rotation of the first and last axes will be opposite in sense. If the number of axes is even the first and last axes will rotate in the same direction.

When the axes of a train are not all parallel, as when a pair of bevel or spiral gears is introduced, the direction of the last follower can be determined only by following the motion of each wheel beginning with the first.

119. Number of Axes in a Train.—It follows from Art. 118 that the number of drivers or followers in a train is always one less than the number of axes.

The least number of axes that can be used for a given velocity ratio will depend upon the limits between which the size of the connected wheels may be allowed to vary.

Let W be the diameter in inches or the number of teeth in the largest wheel of a train, while V represents the same properties of the smallest allowable wheel. Let the large wheels be the drivers and the pinions the followers.

The least number of axes will in this case be obtained when each driver contains W teeth and each follower V teeth. If M be the number of axes we shall have $M-1$ followers and $M-1$ drivers.

Let the value of the train, $\frac{\alpha_m}{\alpha_1}$ be represented by ε.

$$\text{Then } \varepsilon = \frac{W_1}{V_1} \times \frac{W_2}{V_2} \times \frac{W_3 \text{ to } (M-1) \text{ factors}}{V_3 \text{ to } (M-1) \text{ factors}}.$$

$$\therefore \varepsilon = \left(\frac{W}{V}\right)^{(m-1)}$$

whence, $\log \varepsilon = (M-1)(\log W - \log V)$
from which

$$M = 1 + \frac{\log \varepsilon}{\log W - \log V}.$$

120. Examples of Simple Trains.—Fig. 108, which illustrates a clock train, is a good example of the use of a train to effect a great change in velocity. The weight W, suspended to the drum L, may be regarded as the driver and B and C, the wheels which move the minute and hour hands are the followers. E is an escape-wheel connected to the pendulum. The pieces L, E, A, B form one train by which the minute hand is driven at a velocity of one revolution per hour. L, E, D, C forms another train which drives the hour hand at a speed of one revolution in 12 hours.

B and C are mounted on the same axis, but the hub of C forms a sleeve which fits loosely over that of B so that they turn independently of each other.

121. Back Gearing of Lathes.—In Fig. 109, *A* is a lathe cone, *B* is a spindle, parallel to that of the headstock, and having fastened to it the gear *L* and pinion *M*. The pinion *K* is fastened to the cone *A* and turns with it. *N* is fitted with an adjustable nut, so that it can be locked to the cone, or revolve with the spindle independent of the cone. *K* drives *L*, and *M* drives the main spindle through the gear *N*, when the latter is unlocked.

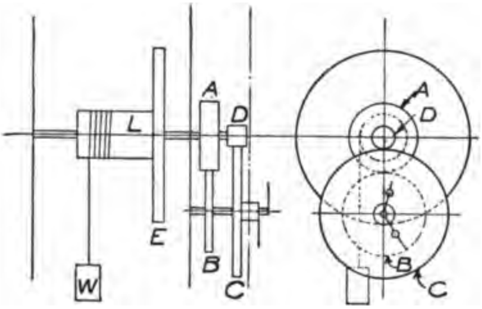


Fig. 108.

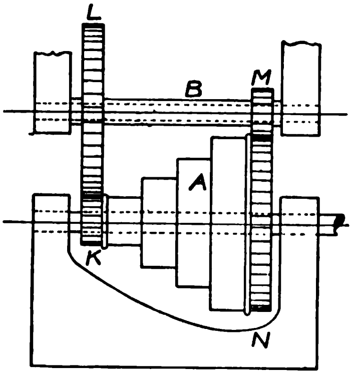


Fig. 109.

The effect of the arrangement is to give three additional speeds to the main spindle corresponding to the steps of the cone. The back gears ought to be so designed that the speed ratios will change by equal intervals.

122. Example of Back Gearing.—Suppose the countershaft in Fig. 110 makes 150 revolutions per minute and the steps on the cones are 10, 8 and 6 in. in diameter, then the speeds of the lathe spindle will be

$$\left\{\frac{E}{H}=\frac{6}{10}\right\}\times150=90 \text{ revolutions per minute}$$

$$\left\{\frac{D}{G}=\frac{8}{8}\right\}\times150=150 \text{ revolutions per minute}$$

and

$$\left\{\frac{C}{F}=\frac{10}{6}\right\}\times150=250 \text{ revolutions per minute}$$

Now let the pinions M and N be 5 in. in diameter and the gears L and P be 12 in. in diameter, then the fastest speed with the back gears in mesh will be

$$\frac{C}{F}\times\frac{N}{P}\times\frac{M}{L}\times150$$

or

$$\frac{10}{6}\times\frac{5}{12}\times\frac{5}{12}\times150=43+$$

and the other two speeds will be

$$\frac{8}{8}\times\frac{5}{12}\times\frac{5}{12}\times150=26+$$

and

$$\frac{6}{10}\times\frac{5}{12}\times\frac{5}{12}\times150=15+$$

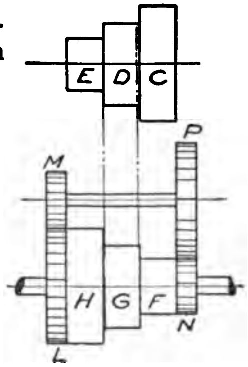


FIG. 110.

Then the ratio of these speeds to each other will be, starting with the highest,

$$\frac{250}{150}=1.66+$$

$$\frac{150}{90}=1.66+$$

$$\frac{90}{43}=2.09+$$

$$\frac{43}{26}=1;65+$$

and

$$\frac{26}{15}=1.73+$$

The change, when the back gears are first thrown in, is a little too large, otherwise the speeds decrease by nearly uniform steps.

123. In some recent designs of high-powered lathes, the step cone is eliminated and the speed changes obtained at the countershaft and by using double sets of back gears. These back gears may be arranged as in Fig. 111, where *A* is the main spindle and *B* the spindle of the back gears. The double gear *EF* slides on a spline so as to make contact with either *C* or *D*. *L* meshes with *M* in the usual way. The velocity ratio is either $\frac{C}{E}\times\frac{L}{M}$ or $\frac{D}{F}\times\frac{L}{M}$.

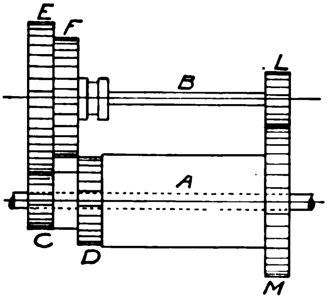


Fig. 111.

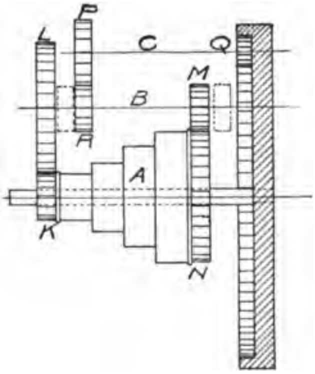


Fig. 112.

Triple back gearing is sometimes arranged as in Fig. 112, where *A* and *B* are the main and back gear spindles as before. When the driving is done through *KL* and *MN*, the arrangement is like the ordinary double gearing. *C* is a third spindle carrying the pinions *P* and Q. The pinion *M* and the spindle *C* are so adjusted that when *M* slides out of contact, *P* and *Q* slide into contact and the spindle is driven through *K*, *L*, *R*, *P* and *Q*. The pinion Q makes contact with an internal gear on the face plate.

One advantage of this scheme is a very slow speed for high powers, another is that the driver Q is very near to the cutting tool.

124. Screw Cutting Trains.—Lathes which are used for cutting screw threads are fitted with trains of gears especially for that purpose. These trains have been designed in a variety of forms by different makers, but the basis for all such designs is shown in Fig. 113.

A is a lead screw to which the cutting tool T is clamped by means of an adjustable lock nut. L is a gear on the lead screw. I is an idler. S is a stud which carries two gears, S' and S_2. P_1 and P_2 are idle pinions, mounted together on an adjustable arm or tumbler and so arranged that S can be driven through them in either direction. H is a gear on the main spindle of the lathe which drives the train.

Suppose the lead screw and the screw to be cut are both right hand, then the operation of the train will be as follows: H drives S' through P_2, S_2 drives L through the idler, I. The rotation of H being clockwise, L will also rotate clockwise and T will move from right to left.

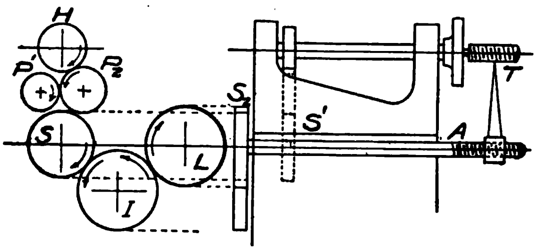


FIG. 113.

If a left-hand thread is to be cut the tumbler is shifted so that H drives S' through P_2 and P', the motion of the succeeding wheels is reversed so that L rotates counter-clockwise and T moves from left to right.

125. Change Gears.—When the screw to be cut does not have the same number of threads per inch as the lead screw, the gear L is changed so that the ratio between the revolutions of the spindle and lead screw will equal the desired ratio between the threads. Thus, if L has twice as many teeth as S_2 and the ratio of S_2 to H is unity, the spindle will turn twice as fast as the lead screw and the screw which is being cut will have twice as many threads per inch. Or, if N equals the number of threads to be cut, and N' equals the threads per inch on the lead screw, while S and L equal the teeth on stud and screw gears respectively, then

$$N = N'\frac{L}{S} \tag{1}$$

If the ratio between the spindle and stud gears is some number Y other than the unity, then (1) must be multiplied by this number or

$$N = N'\left(\frac{L}{S}\right)Y \tag{2}$$

Suppose it is required to cut a screw having eight threads per inch.

If the lead screw has 6 threads, the stud gear 24 teeth, and the ratio between stud and spindle is unity, then from (1):

$$6\left(\frac{L}{24}\right) = 8$$

$$\text{and } L = 32$$

If the stud gears have twice as many teeth as the spindle gear, the ratio is 2 and the formula becomes,

$$12\left(\frac{L}{24}\right) = 8$$

$$\text{from which } L = 16$$

126. Gear Table.—Every screw-cutting lathe has a set of change gears and a table which gives the number of teeth required on the lead screw and stud gears in order to cut a given number of threads. Fig. 114 is the table from a Reed lathe. Column 1 gives the number of threads to be cut. No. 2 is the number of teeth on the stud gear, and No. 3 the number of teeth on the lead-screw gear.

Suppose it is required to cut a screw of seven threads per inch. The stud gear will have 48 teeth and the screw gear 56 teeth. The table shows that the stud gear does not have to be changed for every different thread, a practical convenience in operating the lathe.

For a cut of 6 threads the stud and screw gears are both shown to have 44 teeth, which means that the lead screw has 6 threads per inch.

127. Compounding.—The latter part of the table, Fig. 114, gives the size of gears for cutting screws with a large number of threads. This is done by using a so-called compounding gear. Fig. 115 is the same train as Fig. 113 with the addition of the compounding gear I on the same stud with the idler. The relation $N=N'\left(\frac{L}{S}\right)$ must be multiplied by the ratio between S and C.

Cut	*Stud*	*Screw*	*Compound*		
3	*48*	*24*	*Cut*	*Stud*	*Screw*
4	"	*32*	*20*	*24*	*40*
5	"	*40*	*22*	"	*44*
6	*44*	*44*	*24*	"	*48*
7	*48*	*56*	*26*	"	*52*
8	"	*64*	*28*	"	*56*
9	"	*72*	*30*	"	*60*
10	*24*	*40*	*32*	"	*64*
11	"	*44*	*36*	"	*72*
12	"	*48*			
13	"	*52*			
14	"	*56*			
15	"	*60*			
16	"	*64*			
18	"	*72*			

Fig. 114.

In the example of Art. 125 if the compound gear has half as many teeth as the stud-gear, equation (1) becomes, $N=6\left(\frac{32}{24}\right)2=16$, that is there will be 16 threads instead of 8.

The use of the compound gear has two advantages: (a) It makes it possible to cut an additional screw by simply throwing in the lever of the compound gear without changing the screw gear (b) Screws with a large number of threads can be cut without using excessively large gears on the lead screw or stud.

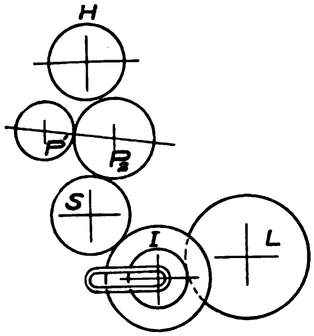


Fig. 115.

128. Nests of Gears.—In some recent designs, the change gears of a lathe are arranged in a "nest," and the speeds necessary to cut different screws are obtained by simply moving a knob on the outside of the machine.

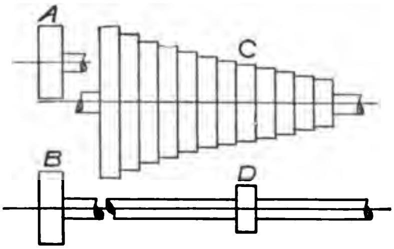


Fig. 116.

Fig. 116, taken from a Lodge & Shipley lathe, shows such a nest of gears. *A* and *B* correspond to the stud and screw gears of an ordinary lathe, *C* is a nest of twelve gears on the lead screw and *D* is a sliding pinion which will engage with any one of the nest.

No changing of gears is necessary, the operator determines

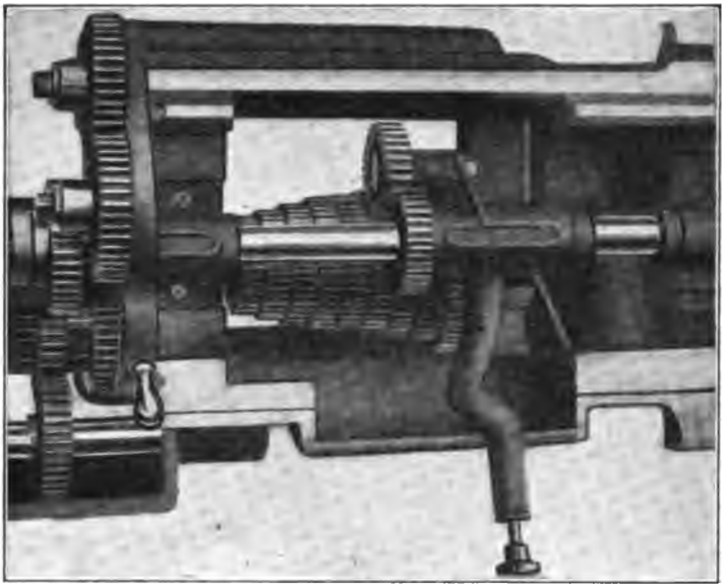


FIG. 117.

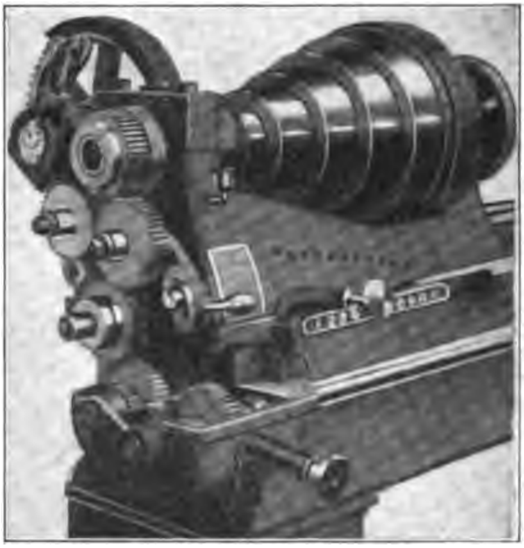


FIG. 117a.

from an attached table the position of the knob for the number of threads to be cut and then places it in a hole over the nest, which corresponds to such number. This places the sliding pinion *D* in mesh with the proper gear of the nest. Fig. 117 is the head end of such a lathe, with the nest, knob and connecting train.

These lathes are sometimes designed with several trains, giving different ratios between *A* and *B* making it possible to cut a large number of threads.

129. The Index Train.—An index is an apparatus for accurately dividing the periphery of a piece of work into a given number of parts, as in cutting gear teeth. It is commonly used on the milling machine.

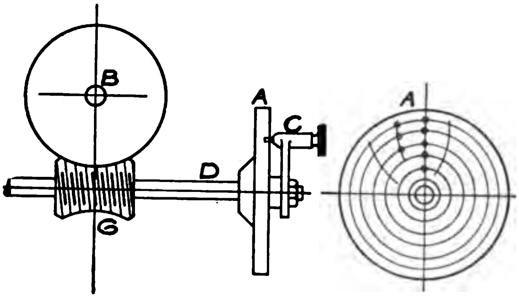


FIG. 118.

Sometimes the index plate is placed directly on the axis of the work to be divided, but the usual method is to put the index plate on a separate spindle and connect it with the work spindle by a train of gears as shown by Fig. 118. *A* is the index plate and *B* is the work spindle the revolutions of which are to be subdivided. At *C* is an adjustable latch pin which is keyed to the spindle *D* and which is made to fit any hole in the index plate. *G* is a worm which meshes with a worm wheel on *B*.

Let *T* equal the number of teeth on the worm wheel, *L* the number of divisions on the periphery of the work and *N* the number of turns to be made by the latch pin.

If the worm is single pitch, then one tooth of the wheel *B* will pass the line of centers for each complete revolution of the latch pin.

If the work is to have the same number of divisions as the worm wheel has teeth then:

$$N=\frac{L}{T}=1.$$

If the number of divisions on the periphery of the work is to be less than the number of teeth on the worm wheel, then

$$N=\frac{L}{T}\text{ is greater than 1.}$$

If the number of divisions on the periphery of the work is to be greater than the number of teeth on the worm wheel, then

$$N=\frac{L}{T}\text{ will be less than 1.}$$

The index plate is laid out with a number of concentric rows of holes, each row having a different number so that fractions of a turn of the latch pin can be indicated with accuracy.

130. Sprocket Wheel Trains.—When gear wheels are connected by a chain the velocity ratio is the same as if they were in direct contact, the chain simply acts as a connector from tooth to tooth.

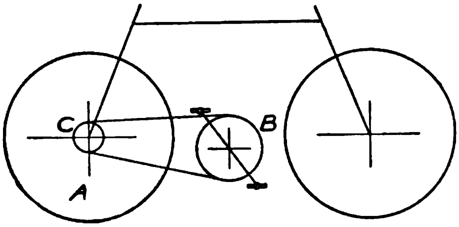


Fig. 119.

In the safety bicycle the sprocket train is geared up so that for a single turn of the pedal crank the driving wheel has a rim velocity equal to that of a wheel of much larger diameter. Thus in Fig. 119, suppose the wheel *A* to be driven from the crank sprocket wheel *B*, through the chain and the small sprocket *C*. For illustration let *A* be 30 in. in diameter and let *B* have 27 teeth and C 9 teeth. Then C and its connected wheel A will make three times as many revolutions in a given time as the pedal crank and the rim velocity of *A* will be equivalent to a wheel 90 in. in diameter and having the pedal directly on its own axis.

Such a bicycle would be rated as a 90-gear machine.

131. Differential Trains.—The principle involved in all trains of this class is that of a follower influenced by two forces, one one impelling it forward and the other backward. The result is the production of velocity ratios that would not be possible with simple trains without a large number of axes or inconveniently large wheels.

Fig. 120 is a form of differential train that is used to transmit motion from the drive shaft to the rear axle of an automobile.

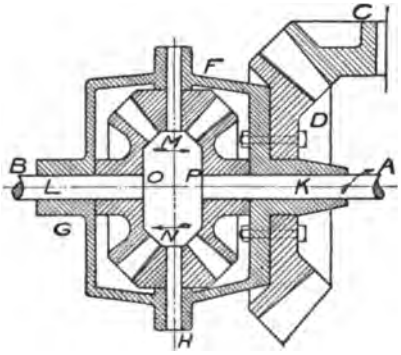


Fig. 120.

A and *B* are the axles and *C* is a bevel pinion on the end of the drive shaft which receives motion from the motor.

D is a gear fastened rigidly to the case *FGH* and turning with it. The case rotates freely on the bearings *K* and *L* of the wheel axles. The pinions M and *N* rotate freely on pins driven into the case at *F* and *H* and serve to lock the gears *P* and *O* so that when the machine is moving in a straight line, the gear *D*, the gear case, the pinions *M* and *N* and the gears *P* and *O* all rotate together as one piece.

Suppose now that the machine is turned to the left, then the left-hand wheel axle must turn slower than the right-hand axle otherwise the left-hand wheel will slip or "skid" on the ground. This means that the pressure on the teeth of *O* will be greater than that on the teeth of *P*. This unbalanced pressure will be exerted on the left side of pinions *M* and *N* and the latter will begin to rotate in opposite directions as shown by the arrows and this counter rotation will accelerate *P*.

If *P*, *O*, *M*, and *N* are all the same size then the number of revolutions of *O* will equal the number of revolutions of the case minus that of *M* or *N*, and the revolutions of *P* will equal the revolutions of the case plus that of *M* or *N*.

132. The Epicyclic Train.—This is a differential train in which one member revolves around another in such a manner that points on the revolving piece describe epicycloidal curves. The simplest form of the train is shown in Fig. 121, where *A* is a gear wheel centered at *O* and *C* is a train arm also centered at *O* and having mounted upon it a second gear *B*, arranged so as to mesh with *A*.

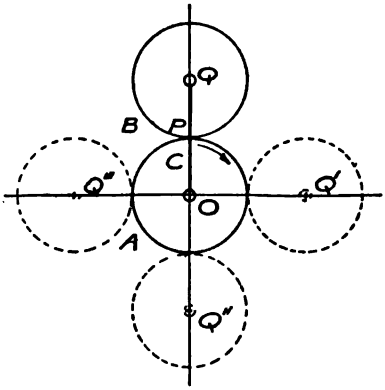


FIG. 121.

If *C* be fixed in position while *A* and *B* both rotate, the arrangement will be a simple train but if either *A* or *B* be fixed while the train arm carrying the other wheel, rotates, the arrangement will be an epicyclic train.

Let *A* be the fixed wheel and let *C* rotate in the clockwise direction, carrying *B* with it. Then, supposing the wheels to have the same number of teeth, for every rotation of the train arm about *O*, *B* will make one complete rotation about the axis *Q*, also in the clockwise direction, solely because of its motion about *O*.

Again let *B* and *C* rotate together as before and let *A* rotate simultaneously and in the same direction. Then the clockwise rotation of *B* will be canceled by the motion of *A*, and *B* will have no absolute rotation on its own axis.

Again let *A* rotate with the same velocity as the train arm but in the counter-clockwise direction. Then the rotation of *B* will be equal to the rotation of *A* plus the rotation of the train arm. That is, for one rotation of *A* and the train arm, *B* will have two complete rotations on its own axis in the clockwise direction.

When *C* is fixed, *A* and *B* rotate in opposite directions and the velocity ratio is said to be negative. If any number of wheels be placed between *A* and *B*, the value of the velocity ratio will be negative if with the train arm fixed, the rotation of *A* and *B* is opposite in direction, that is, if the number of axes is even. If the number of axes is odd, *A* and *B* will rotate in the same direction and the value of the velocity ratio will be positive.

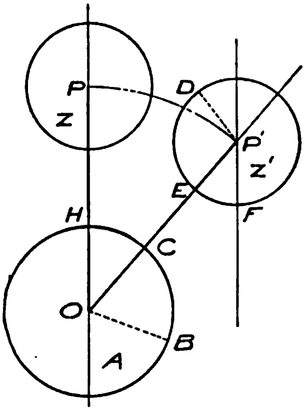


FIG. 122.

133. Velocity Ratio of Epicyclic Trains.—If the number of teeth on *A* and *B*, Fig. 121, is not the same, the absolute rotation of *B* will equal the rotation of the train arm plus or minus the ratio of the number of teeth on *A* and *B*. Also the rotation of either wheel with respect to the train arm, is equal to the absolute rotation of the wheel plus or minus the rotation of the arm.

In Fig. 122, let *OP* be a train arm and let *A* and *Z* be the first and last wheels of a train. Let *L*, *M* and *N* represent the rotations of the arm, the first and last wheels respectively, and let *X* represent the velocity ratio *Z* to *A*, that is, the value $\frac{\alpha_m}{\alpha_1}$ of Art. 117.

Suppose the arm *OP* to turn to some new position as *OP'* while wheel *A*, driven by some outside agency, rotates through the angle *HOB*, driving *Z* through intermediate gears through the angle *DP'F*. Then the angle *HOB* is to the angle *DP'F* as the number of teeth on *Z* is to the number on *A*. Also *HOC* equals $EP'F$ and in this case $(HOB-HOC)$ is the rotation of A relatively to the arm. Similarly $(DP'F-EP'F)$ is the rotation of Z relatively to the arm. These relations being true for any angle are true for complete rotations and we may therefore write:

$$X=\frac{N-L}{M-L} \tag{1}$$

This is the general equation for epicyclic trains and by transposing terms we get, for the arm and wheels:

$$L=\frac{MX-N}{X-1} \tag{2}$$

$$M=L+\frac{N-L}{X} \tag{3}$$

$$N=L+X(M-L) \tag{4}$$

If either wheel is fixed M or N will be equal to zero and the formulas will be modified accordingly.

134. Example of the Epicyclic Train.—In Fig. 123, let A be a train arm, B the first and E the last wheel of a train.

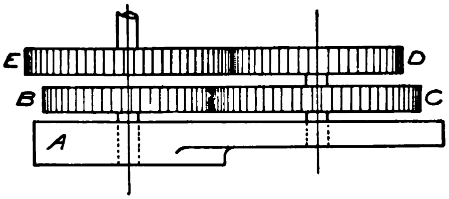


Fig. 123.

Suppose B, driven independently of the arm, to have 100 r. p. m. and to have 99 teeth while the other wheels of the train have 100 teeth each. If the rotation of the train arm is 0,

$$N=100\frac{B}{C}\times\frac{D}{E}=100\frac{99}{100}\times\frac{100}{100}=99.$$

Since there are three axes, X is positive and its value is

$$\frac{99}{100}=0.99.$$

As the drivers and followers here differ by a single tooth, this is the smallest difference in velocity that it is possible to obtain with a train of three axes without increasing the number of teeth on all the gears. By giving the train arm simultaneous rotation, however, it is possible to obtain a much smaller difference in velocity without otherwise changing the train. For example, let E be required to make 999 revolutions while B makes 1000. Then from (1), Art. 133,

$$\frac{N-L}{M-L}=0.99,$$

and from (2),

$$L=\frac{(1000\times 0.99)-999}{0.99-1}=\frac{-9}{-0.01}=900 \text{ r.p.m. in the same direction.}$$

135. Planetary Motions.—Epicyclic trains are sometimes called "planetaries," the name being derived from Watt's "Sun and Planet Motion" which is an epicyclic train having the last wheel, N, fixed. The train was originally applied between the connecting rod and shaft of a beam engine as a substitute for a crank.

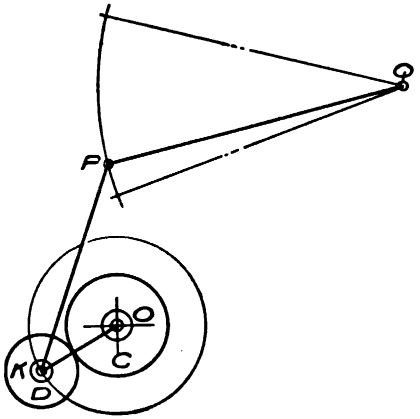


FIG. 124.

In Fig. 124, QP is one end of the walking beam, C is a spur wheel on the main shaft and D is another spur wheel attached to the connecting rod and gearing with C. OK is a link which holds

C and *D* in contact. As *QP* oscillates *OK* and *D* rotate around the center *O*. Considered as an epicyclic train, *OK* is the train arm, *C* is the first wheel and *D* is the last.

As there are but two axes the value of the train is negative and when *C* and *D* have the same number of teeth, $X=(-1)$. *D* has no absolute rotation on its own axis, hence by putting $N=0$, and substituting values in formula (3) of Art. 133.

$$M=L\left(1-\frac{1}{-1}\right)=2L$$

That is, for one revolution of the train arm, corresponding to one cycle of the beam *QP*, the shaft makes two revolutions instead of one.

If *C* has twice as many teeth as *D*, then $X=(-2)$ and,

$$M=L\left(1-\frac{1}{-2}\right)=\frac{3}{2}L$$

That is, the shaft revolves three times while the end of the beam makes two cycles.

PROBLEMS

1. A shaft which makes 100 r.p.m. is connected to a third shaft through a pair of pulleys 12 and 15 in. in diameter, and a pair of gears having 100 and 150 teeth. What is the speed of the third shaft?

2. The drum of a hoist is 10 in. in diameter. On the drum shaft is a 6 in. pinion which is driven by an 18 in. gear fixed to a shaft that makes 25 r.p.m. How many feet of rope per minute will be wound on the drum?

3. What is the direction of rotation in Prob. (1) if the first shaft turns clockwise?

4. A lathe counter-shaft makes 150 r.p.m. The lathe and counter-shaft cones have four steps of 10, 8½, 6½ and 5 in. diameter. If the ratio of the back gearing is 10 to 1, find the spindle speeds.

5. A lathe counter-shaft makes 160 r.p.m. The ratio of the back gears is 12 to 1. Design cones with four steps such that the speed ratios will be uniform and about 2 to 1.

6. A set of double back gears are arranged as in Fig. 111. Find the spindle speeds if the counter-shaft makes 125 r.p.m. and the back gears have teeth as follows: *M* and *E* 90, *L* and *C* 14, *D* 20, and *F* 84.

7. The lead screw of a lathe has 6 threads per inch. The ratio between the spindle and stud is 3 to 1. Find change gears to cut right-hand threads of 4, 6, 8, 9 and 12 pitch.

8. In Prob. 7 suppose a compounding gear having a ratio of 2½ to 1 is

thrown in, show the number and pitch of threads that can be cut with the same change gears.

9. A right-hand lead screw has 8 threads per inch. Make a gear table similar to Fig. 114 to cut threads from 2 to 26.

10. Show how to set the index pin on Fig. 118 to cut a gear of 30 teeth. Worm wheel has 40 teeth.

11. A bicycle has 34-in. wheels, the small sprocket has 12 teeth and the large one 30 teeth. What is the "gear" of the wheel?

12. The motor shaft of an automobile makes 900 r.p.m. The driving sprocket has 9 teeth and the axle sprocket, 27 teeth. If the wheels are 38 in. in diameter, how many miles per hour will the machine travel?

13. In Fig. 120, suppose C makes 1000 r.p.m. and the velocity ratio of C to $D = \frac{4}{1}$. If the car is turning a corner so that O turns one-half as fast as P, what will be the speed of each driving wheel in r.p.m.

14. In Fig. 123, if the train arm makes 20 r.p.m. and B, C, D, and E have 99, 101, 98 and 100 teeth find the value of N, if B is fixed and revolves with the arm.

15. Find number of teeth on each wheel of an epicyclic train of 4 axes, such that when the arm revolves 5 times the last axis will revolve 3 times. First wheel being fixed to the arm.

16. Design an epicyclic train of spur gears of not over 100 teeth to transmit a velocity ratio between the first and last wheel of $\frac{1}{100}$.

17. A steam engine (Fig. 1) fly-wheel drives a belt to a counter-shaft and this in turn is geared to a large engine-lathe. The engine makes 150 r.p.m., the wheel is 15 ft. in diameter and the stroke of the engine is 24 in. Find average speed of piston in feet per minute.

18. Find angular and linear velocity of fly-wheel rim.

19. Find linear velocity of center of crank pin, and from this the velocity of piston when crank is at 45 degrees, the connecting rod being 72 in. long.

20. In same way find velocity of piston when crank is at 135 degrees.

21. If counter-shaft is 24 ft. from engine shaft and makes 270 r.p.m., find diameter of small pulley and length of open belt to connect the two.

22. If lathe spindle makes twice as many revolutions as stud gear and lead screw has two threads per inch, find gears for stud and screw to cut 1, 1.5, 2, 2.5, 3, and 17 threads per inch. No gear smaller than 24 teeth.

23. Find speed of carriage in cutting 2 threads per inch.

CHAPTER VIII

DIRECT CONTACT

TRANSMITTING MOTION BY PURE ROLLING

136. Friction of Contact Surfaces.—Whenever two surfaces are in contact and there is a normal pressure between them, there will be a resistance to the sliding of one surface on the other and this resistance we call friction.

All surfaces are covered with projections greater or smaller, according to the degree of smoothness. When the two surfaces are brough into contact the projections interlock. If pressed together with great force the friction will be greater with smooth surfaces than with rough ones because there are more projections to interlock. On the other hand if the pressure is not very great the friction will be greater with rough surfaces. The use of very smoothly finished friction clutches is an illustration of this principle.

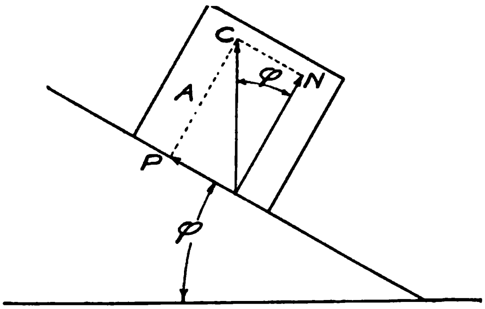


FIG. 125.

When oil is used between two contact surfaces it prevents the projections from interlocking or as we say it decreases the friction.

Perfectly smooth surfaces are not attainable, the friction is always greater than zero and varies with the substances in contact. A factor called the coefficient of friction has been found experimentally for the ordinary materials of engineering.

In Fig. 125, A is a block, the weight of which acts vertically, downward through its center of gravity. The block rests on a plane of the same material and both are uniformly rough. If the plane be tipped up through some angle as φ the block A will slide down. φ, the angle at which the block begins to slide is called the angle of repose. Tan. φ is the coefficient of friction and is equal to $P \div N$, where P is the force due to the friction or the resistance to sliding produced by the roughness of the surfaces. N is the normal pressure between the surfaces. The resultant of P and N is equal to the weight, C, of the block.

137. Friction Wheels.—When rolling contact is transmitted by friction wheels the motion of the follower is not positive. In other words, if the friction is not great enough the surfaces will slip and the follower will stop moving. Such slipping will occur whenever the tangential force required to turn the follower is greater than the frictional force P, between the surfaces. In Fig. 126 let B be the driver and suppose PT to be the tangential force required to rotate A.

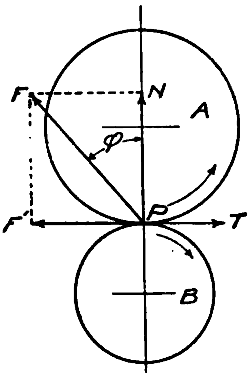


FIG. 126.

Take from a table the value of tan φ for the materials in contact and lay off the angle φ from the line of centers as shown. Lay off PN, equal to the normal pressure between the wheels and complete the triangle PFN.

Then $PF' = NF =$ the friction.

If B is to drive A with no slipping, PF' must be greater than PT, or expressed algebraically:

$$PF' = FN = PN \tan \varphi > PT.$$

138. Cylindrical Friction Wheels.—A and B, Fig. 127, represents two rolling cylinders or friction wheels in contact along elements at P. The rules of Chapter III for contact pieces hold true for these wheels for:

(a) From the nature of their curved outlines, the point of contact P always remains on the line of centers so that if any motion is transmitted it must be by rolling contact.

(b) The common normal to the curves at *P* coincides with the line of centers and passes through *O* and *O′*, therefore the motion of the follower is not positive.

(c) Since the point of contact *P* lies between *O* and *O′*, the wheels will rotate in opposite directions.

(d) Since by rule III of Art. 40, the velocity ratio is inversely proportional to the radii, this ratio must be constant since the radii are constant.

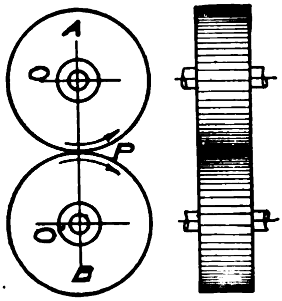


FIG. 127.

139. Grooved Friction Wheels.—It follows from Art. 137 that as the tangential force necessary to drive a friction wheel increases, the normal pressure must also be increased, or the driver will slip over the follower without moving it. Any increase in the normal pressure increases the load, and consequently the wear, on the wheel bearings. Grooving the wheels as in Fig.

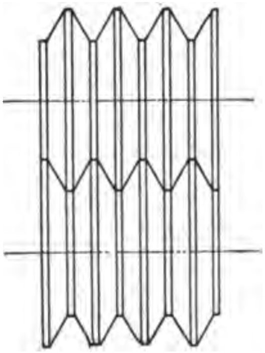


FIG. 128.

128, decreases the pressure at the bearings in two ways: (a) By increasing the frictional area, so that the wheels have more driving power; (b) by changing the direction of the pressure between the frictional surfaces.

The objection to grooved wheels is that the action is no longer pure rolling contact. This is illustrated by Fig. 129 where the wheels *A* and *B* may be assumed to roll in contact along circles which pass through *P* about midway of the grooves. Points on these circles may have the same linear velocity for both wheels, but points on either *A* or *B* which are outside of this line of equal velocities, move faster than points which are on it, and points inside move slower than points that are on it, their velocities being in direct ratio to their distance from the centers. Therefore, relatively fast moving points on one wheel come in contact with relatively slow moving points on the other, and the result is a grinding action which increases the friction and wear without doing any useful work.

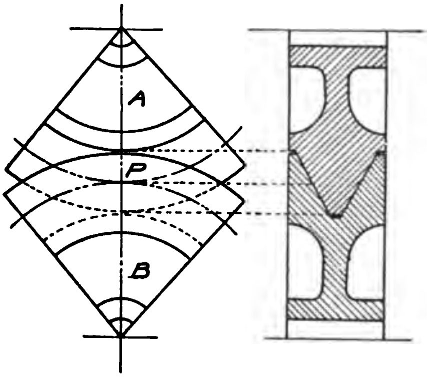


Fig. 129.

140. Angle of Pressure, Grooved Friction Wheels.—The effect of changing the angle of pressure by using grooved surfaces is shown by Fig. 130.

Let F = the pressure at either bearing;

θ the angle between the sides of the groove and the line of centers;

N and N' the normals at the middle of the grooves.

The force F produces a number of smaller parallel forces which act perpendicular to the sides of the grooves and the resultant of these forces may be assumed to act at N and N'. Produce N and N' until they intersect at P. The force F may then be resolved at P into two components R and R'.

From the figure we have:

$$R \sin \theta = \tfrac{1}{2}F \text{ or } R = \frac{F}{2 \sin \theta}$$

also

$$R' \sin \theta = \tfrac{1}{2}F \text{ or } R' = \frac{F}{2 \sin \theta}$$

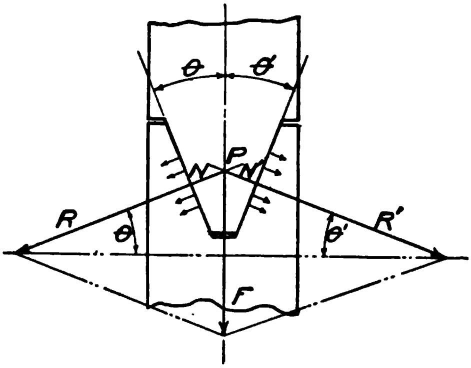


Fig. 130.

Then $R = R'$ and $R + R' = \frac{F}{\sin \theta}$ = the sum of the small parallel forces produced by F.

For any value of θ less than 90 degrees, $\sin \theta$ is less than unity and $R + R'$ is greater than F.

When $\theta = 90$, $R + R' = F$.

As θ approaches 0, F decreases in proportion to $R + R'$. If the groove is made too pointed the pieces will wedge together and transmit no motion. Practice has shown the best results when $\theta + \theta'$ is made from 40 to 50 degrees.

52. Brush Wheels.—Fig. 131 shows a special form of cylindrical friction wheels. The small wheel B is so mounted upon its axis that it may slide either way across the friction plate A. When B is located near the edge of A, contact between the surfaces approximates pure rolling. When *B* is placed near the center of *A* the contact is less satisfactory because there is a greater tendency to rotate *B* in the plane of its axis, owing to the different directions of the contact points of the two pieces. The velocity ratio of this mechanism varies with the distance from *B* to the center of *A*.

The directional relation may be reversed by changing *B* to the right or left of the center of *A*.

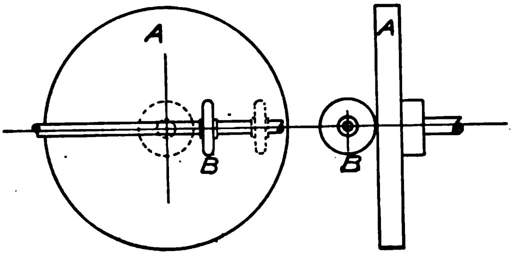


Fig. 131.

This is a convenient device when the velocity or direction of a follower has to be frequently changed.

142. Conical Friction Wheels.—If *AO* and *BO*, Fig. 132, are the axes of two cones which are in contact along the element *TO*, their base circles are similar to the outlines of a pair of cylindrical friction wheels, and they will roll together with a velocity ratio depending upon their radii the same as if they were in a single plane. The same thing is true of any other

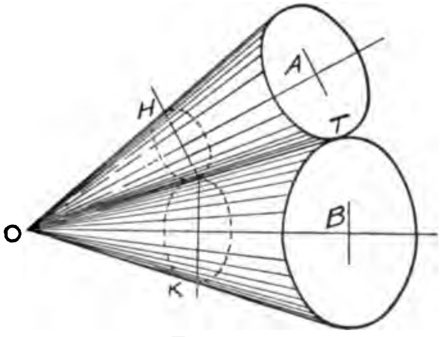


Fig. 132.

pair of sections as *H* and *K*, between the base and the apex. Hence it is true of the frustra between the sections. The frustra of cones may therefore be used to transmit motion between intersecting axes and the conditions of contact, and velocity ratio will be the same as for cylinders.

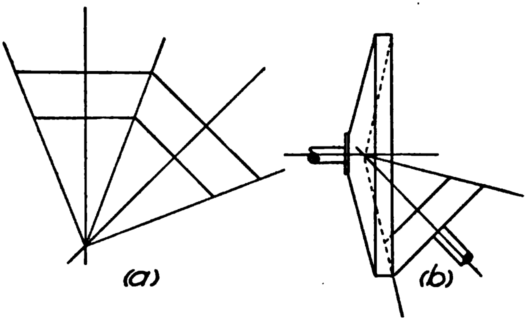


FIG. 133.

Fig. 133 shows the cones in external and internal contact. As in the case of cylinders, the axes may be made to rotate in the same or in opposite directions by placing them on the same or on opposite sides of the point of contact.

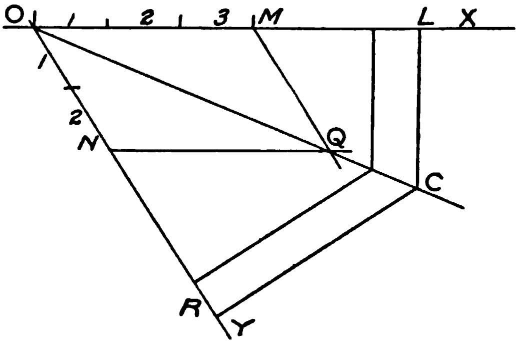


FIG. 134.

143. To Lay out a Pair of Friction Cones.—Let the angular velocity ratio $\frac{\text{Driver}}{\text{Follower}} = \frac{3}{2}$; the angle between the axes being 60 degrees, Fig. 134. If *OX* is the driver and *OY* the follower axis, take any convenient unit of length and lay it off three times along *OX*, and twice along *OY*, locating the points *M* and *N*. Through *M* draw a line parallel to *OY*, and through *N* draw a line parallel to *OX;* these lines will intersect at *Q*, and a line drawn through *OQ* will be the common element of the cones. Now an indefinite number of pairs of cone frustra, having the common element *OQ*, will transmit the given velocity ratio. The diameter of one or both of the cones must be known in order to find their position on the axes. Suppose the radius of the driver is to be 2 in., locate *L* so that the perpendicular *CL* is 2 in. long, then through *C* draw *CR* perpendicular to *OY*. The point *L* can be found quickly on the drawing board by holding the edge of a scale perpendicular to *OX* and sliding it out from *O* until the angle *COX* subtends a length of 2 in. on the scale.

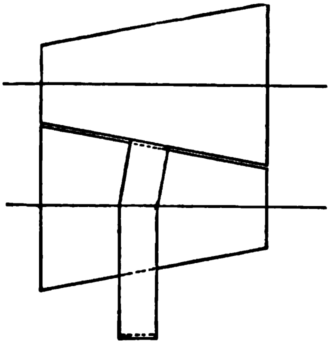


FIG. 135.

144. Friction Cones for Variable Velocity Ratio.—A method of using friction wheels in which the velocity ratio of the axes may be varied at pleasure within certain limits is shown by Fig. 135. A space is left between the cones and a leather belt hung loosely over the lower one. By sliding the belt along the cones contact may be made at any part of their surfaces and the velocity ratio will vary as the radii of contact vary.

Fig. 136 shows another method of accomplishing the same result. A small rotating disk *C* is made to slide along a shaft and make contact between different parts of the cone surfaces. In both of these cases the velocity ratio is a minimum when contact is made with the smaller end of the driver and a maximum when contact is made with the larger end of the driver.

145. Hyperbolic Friction Wheels.—Cylindrical friction wheels connect parallel axes, the surfaces being generated by the rotation of an element of the surface about the axis. Conical friction wheels connect intersecting axes, the conical surfaces being generated by the rotation of an element of the surface about the base of the cone, using the apex of the cone as a center of rotation. When two axes are neither parallel nor intersecting, they may be connected by wheels having hyperboloids of revolution for their contact surfaces.

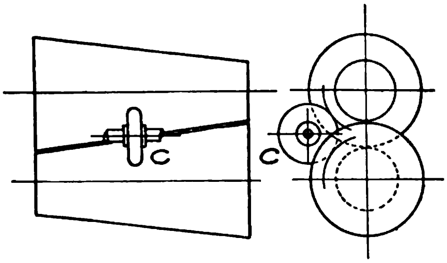


FIG. 136.

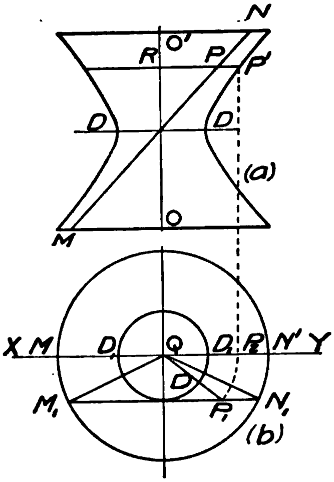


FIG. 137.

A hyperboloid of revolution is a warped surface which is generated by revolving one straight line about another straight line which is not in the same plane with the first. The surface is shown in vertical projection at Fig. 137 (*a*) and in horizontal projection at Fig. 137 (*b*) the corresponding projections of the axis being OO' and Q. MN and M_1N_1 are the projections of an element. As MN revolves about the axis of the surface, any point on it, as P, describes a circle, the radius of which is projected vertically at RP and horizontally, in its true length at QP_1.

Any number of such points may be taken on *MN* and the circles described by them will vary in size, the smallest, having the radius *QD* and being vertically projected at *DD*, is called the gorge circle. The circle described by the outer ends of the element form the bases of the hyperboloid.

The circle described by the rotation of *P* will cut the axis *XY* at P_2 and this point will be projected vertically at *P′*. If other points between the gorge circle and the bases be found in a similar way, a curve drawn through them will be a hyperbola. The surface may then have a second generation by the revolution of this curve about the axis *OO′*.

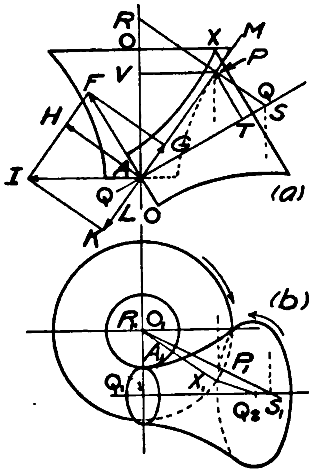


FIG. 138.

When two hyperbolic friction wheels are in contact, their line of contact will be the common element *MN* which is also the common generatrix of the two surfaces.

146. Laws of Contact for Hyperbolic Friction Wheels.—The common element of contact may lie between the axes or it may be located on the same side of both axes, as with cylinders and cones. It follows then that the hyperboloids may rotate in the same or in opposite directions.

If a tangent be drawn to the gorge circles of two hyperboloids, the surfaces being in contact along a common element, this tangent will represent the line of action between the two sections which are in contact at the gorge. This tangent will also cut the line connecting the centers of the two gorge circles in a fixed point. These sections will therefore roll on each other with constant angular velocity ratio. The same may be proved of any other pair of contact sections, it is therefore true of the surfaces.

Two contact sections taken through any point on the element of contact have radii of constant length, the driving action is therefore not positive.

At the gorge circles the point of contact will lie on the line connecting the centers of the circles as in *A*, Fig. 138. If we take any other pair of contact radii as Q_2X_1, and O_1X_1, the point of contact will not lie on the line connecting the centers of the circular sections. The contact is therefore pure rolling at the gorge and at any other section it is mixed rolling and sliding and the further the sections are taken from the gorge the greater the amount of sliding becomes.

147. Angular Velocity Ratio of Rolling Hyperboloids.—Fig. 138, let two rolling hyperboloids with axes *OO* and *QQ* be shown in vertical projection at (*a*) and in horizontal projection at (*b*). Let the inclined hyperboloid be the driver and the direction of motion be as shown by the arrows. Let *LM* be a common element of contact and *RPS* a normal to the contact surfaces through any point on the element. Suppose at a given instant a point *A* on the gorge circle of the inclined hyperboloid to have a velocity *AF*, in a direction tangent to that circle, then *AH* perpendicular to *LM* will be the normal component of *AF* and *AG* will be the tangential component. By Art. 43 the normal component at a point of contact represents the transmitted motion and the tangential component represents the lost motion or slipping. If *AI* represents the velocity at the same instant of a driven point on the gorge circle of the follower, its normal component must also equal *AH* by Art. 35. Its tangential component will be *AK*.

Let α = the angular velocity of the driver $= \frac{AF}{A_1Q_1}$,

Let α' = the angular velocity of the follower $= \frac{AI}{A_1O_1}$

$$\text{then} \frac{\alpha}{\alpha'} = \frac{AF}{A_1Q_1} \times \frac{A_1O_1}{AI} = \frac{AF}{AI} \times \frac{A_1O_1}{A_1Q_1} \qquad (1)$$

From the similar triangles *AIF* and *ARS*, $\frac{AF}{AI} = \frac{AS}{AR}$.

From the nature of the contact surfaces the normal *RPS* will intersect the center lines in some points as *R* and *S*. In horizontal projection the normal segments P_1R_1 and P_1S_1 are proportional to the radii of the gorge circles A_1Q_1 and A_1O_1, that is,

$$\frac{A_1O_1}{A_1Q_1} = \frac{P_1R_1}{P_1S_1} = \frac{PR}{PS}$$

Substituting in (1) for $\frac{AF}{AI}$ and $\frac{A_1O_1}{A_1Q_1}$ the values just found,

$$\frac{\alpha}{\alpha_1}=\frac{AS}{AR}\times\frac{PR}{PS}=\frac{AS}{PS}\times\frac{PR}{AR} \qquad (2)$$

Again the triangles APS and PTS are similar by construction,

$$\text{therefore } \frac{AS}{PS}=\frac{AP}{PT}$$

Also the triangles APR and APV are similar,

$$\text{hence } \frac{PR}{AR}=\frac{PV}{AP}$$

Substituting these values in (2) we have finally for the velocity ratio,

$$\frac{\alpha}{\alpha^1}=\frac{AP}{PT}\times\frac{PV}{AP}=\frac{PV}{PT} \qquad (3)$$

That is to say, the angular velocities are inversely proportional to the projections, upon a plane which is parallel to the axes and the common element, of the perpendiculars let fall from any point in the common element to the axes.

148. To lay out a Pair of Hyperbolic Friction Wheels.—In Fig. 139, OO and QQ are axes which are neither parallel nor intersecting. Let it be required to construct upon them a pair of hyperbolic friction wheels such that the velocity ratio $\frac{OO}{QQ}=\frac{3}{4}$.

Project OO and QQ upon a vertical plane parallel to both axes. The projections will intersect in some point A. Supposing OO to be the driving axis, lay off on OO from A, three units of length and from A on QQ lay off four units of length. Through the points X and Y so found draw lines parallel to the axes and through their intersection P, and the point A, draw the line AP. Then AP will be the vertical projection of a common element of the surfaces. Through P perpendicular to AP draw the common normal CPR. The horizontal projection of CPR will be $C'P'O'$. Through P' draw a line parallel to the horizontal projection of the axes, its intersection with the line of centers will locate the horizontal projection of A and determine $A'O'$ and $A'Q'$ the radii of the gorge circles.

Relatively thin laminations of the hyperbolic surfaces are generally used for the wheels and an indefinite number of pairs of laminations can be found which having *AP* for a common element will transmit the velocity required.

The radius of one base and the thickness of one of the wheels must now be assumed.

Suppose the diameter of that base which is farthest from the gorge plane on axis *OO* to be 3 in. in diameter and let the wheel be $1\frac{1}{2}$ in. thick.

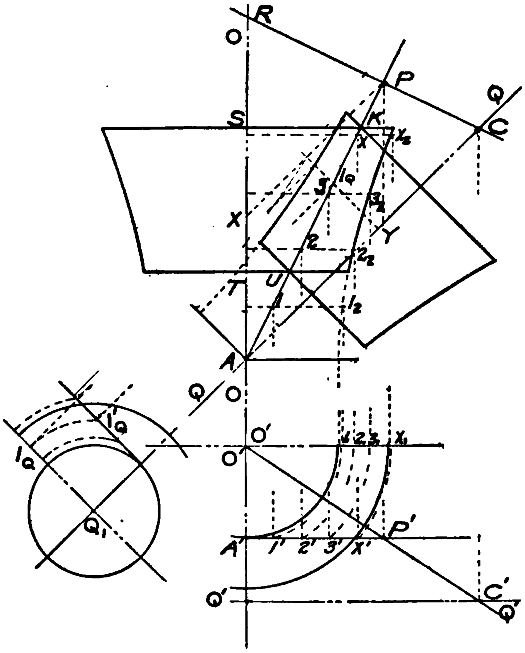


FIG. 139.

With center at *O′* and radius equal to *O′A′*, draw the horizontal projection of the gorge circle and from the same center draw a circle 3 in. in diameter to represent the upper base. Lay off on *A′P′* the points 1′, 2′, 3′, ... *X′*, equal distances apart and project them to *AP* in points 1, 2, 3, ... *X*. With radii *O′*1′, *O′*2′, *O′X′*, cut the horizontal center line through *O′O′*, in points 1_1, 2_1, 3_1 . . . . . X_1, and project these points to 1_2, 2_2, 3_2, ... X_2, on the horizontals through 1, 2, 3,

. . . . . X, thus giving points in the outline of the first hyperboloid. This curve may then be duplicated on the other side of the axis giving the complete outline. The plane of the section whose diameter is 3 in., will cut the common element at K. Through K draw the vertical projection of the upper base of the second hyperboloid, perpendicular to QQ. On any convenient center Q_1, draw the horizontal projection of the gorge circle of this second hyperboloid and then proceed as with the first one to find the outline.

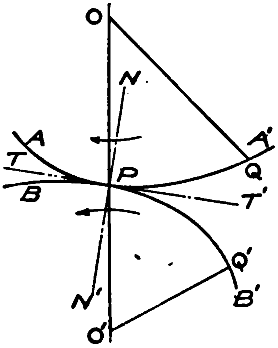


FIG. 140.

Lay off ST equal the thickness of the first wheel. The plane of its lower base will then cut the element at U and the plane of the lower base of the second wheel will be drawn through this point and perpendicular to QQ.

149. Non-circular Wheels.—The laws of contact for wheels with non-circular outlines may be studied on Fig. 140, where O and O' represent the axes of two pieces having the curved outlines AA' and BB'. Let the pieces be in contact at P and let Q and Q' be any other points so taken that the curve PQ is the same length as PQ'.

The pieces will rotate in opposite directions, for the centers O and O' are on opposite sides of the contact point P. (Art. 40, Rule I). The centers O and O' may also be placed on the same side of P, in which case the pieces will rotate in the same direction.

As long as PQ and PQ' are the same length, and points on them have the same linear velocity, the point of contact will remain on the line of centers and the curves will have pure rolling contact, if it departs from this line in either direction, the contact will be mixed rolling and sliding.

The velocity ratio being inversely proportional to the contact radii, if the contact point remains on the line of centers, the contact radii of AA' must increase at the same rate that those of BB' decrease or *vice versa* The velocity ratio will therefore vary from $\frac{OP}{O'P}$ to $\frac{OQ}{O'Q'}$.

The common normal NN' does not pass through either of the fixed centers. With PQ as the driver in the position and direction shown, the driving action is positive.

150. Rolling Ellipses.—Let Fig. 141 represent a pair of equal and similar ellipses, each ellipse rotating about one of its fixed foci as a center.

Since P, the point of contact, lies between the centers, rotation will be in opposite directions. Sections of ellipses may be laid out with P on the same side of the center so that rotation will take place in the same direction but whole ellipses must be equal and similar hence they cannot be constructed for internal contact and rotation in the same direction.

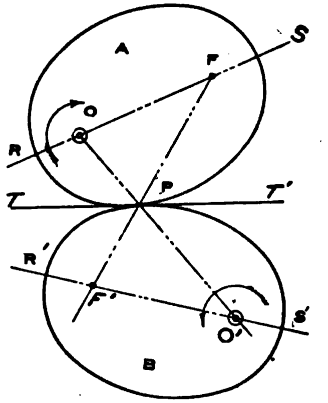


Fig. 141.

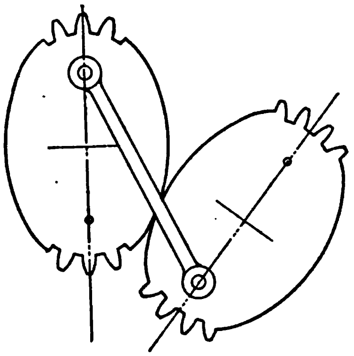


Fig. 142.

If TT' is the common tangent to the curves at the point P, than for any two equal and similar ellipses, in any position of contact, the four angles TPO, $T'PF$, TPF' and $T'PO'$ are all equal.

Then OPF and $O'PF'$ must be equal, and FPF' and OPO' are straight lines.

It follows therefore that P must always lie on the line of centers and the contact be pure rolling.

The velocity ratio will vary between the limits $\frac{OR}{O'R'}$ and $\frac{OS}{O'S'}$, and will have one maximum and one minimum value in each revolution.

The driving action will be positive only during the part of the revolution in which the radius of the driver is increasing. The ellipses may be held in contact when the radius of the driver is decreasing by a link or by placing teeth at the ends of the major axis as shown in Fig. 142. The kinematical relation is then entirely changed as the wheels cease to transmit motion by frictional rolling contact.

151. To Lay out a Pair of Ellipses For a Given Velocity Ratio.—The two equal ellipses, Fig. 143, are laid out so that they will rotate about the fixed foci O and O' and transmit a velocity ratio varying between the limits $\frac{OP}{O'P}$ and $\frac{OR}{O'S}$. For convenience, let these limits have the numerical values of 4 and $\frac{1}{4}$ respectively. In two similar ellipses the sum of any two pairs of contact radii must be equal, so $(OP+O'P)=(O'S+OR)$. From the nature of an ellipse $OF=O'F'$, also OR, FP, $O'P$ and $F'S$ are all equal and the major axes equal the center distance OO'. Draw center line RS, and lay off the center distance OO' equal to $(4+1)$ units. In the position shown, the velocity ratio of A to B is $\frac{O'P}{OP}=\frac{1}{4}$, hence $O'P=1$ and $OP=4$. Then FP and OR each equal 1, and OF, the focal distance, equals 3. The ellipses can now be constructed on these foci, in the position shown, by any of the known methods.

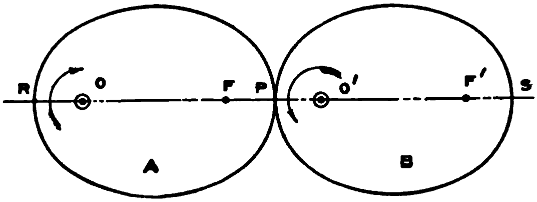


FIG. 143.

If the ellipses are to be laid out in any other position, as that shown in Fig. 141, we make use of the property that any point on the curve has the sum of its distances from the two foci constant, equal to the major axis and equal to the center distance. Lay off RS in the desired position; find OF and construct the first ellipse as described above. Let the curves be in contact at any point *P*. Draw *FP* and *OP* and produce them indefinitely. Lay off $PO' = PF$ and $PF' = PO$. Then *O'* and *F'* are the foci of the other ellipse, *R'S'* is its center line, and either pair of foci, *F* and *F'* or *O* and *O'*, may be the fixed centers of rotation.

152. To Construct a Pair of Non-circular Segmental Wheels.—Fig. 144 shows a method of constructing approximately a pair of segmental wheels which will transmit motion positively for part of a revolution the contact being pure rolling and the velocity ratio varying between fixed limits.

Let *O* and *O'* be the fixed centers of rotation and let *A* be the driver turning through an angle of 60 degrees while the velocity ratio varies from $\frac{3}{2}$ to $\frac{2}{4}$.

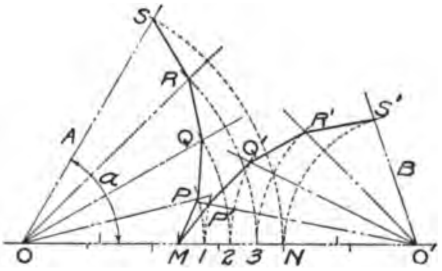


FIG. 144.

Divide *OO'* first into 3+2 equal parts and again into 2+4 equal parts. Then at the beginning of the motion the velocity ratio $\frac{3}{2}$ will equal $\frac{O'M}{OM}$ and at the end of the motion the ratio $\frac{2}{4}$ will equal $\frac{O'N}{ON}$. Divide *MN* into any number of parts, equal or unequal, and then divide the angle α into the same number of corresponding parts. Draw the rays *OP*, *OQ*, etc. With radius *O*1 draw an arc cutting *OP* at *P*. With radius *O'*1, draw an indefinite arc, and then with *MP* as a radius, and center at *M*, cut the latter arc at *P'*. Join *MP* and *MP'* and draw *O'P'*. In the same way, describe arcs with radii *O*2, *O*3, and *ON*, cutting *OQ*, *OR* and *OS*. Join *PQ*, *QR* and *RS*. Next draw arcs with radii *O'*2, *O'*3 and *O'N*, and lay off $P'Q' = PQ$, $Q'R' = QR$ and $R'S' = RS$; join these points and connect them with *O'*.

Then as A and B revolve, the points P and P', Q and Q', etc., will fall together at the line of centers, giving the required velocity ratio.

By taking the points 1, 2, 3, etc., very close together the broken lines can be made to approach true curves by any desired degree of accuracy.

153. Elliptical Sectors.—Sectors of two ellipses can be made to roll together and transmit motion with a velocity ratio that varies between fixed limits, each sector making a partial revolution about one focus. If the angle through which one sector turns and the maximum and minimum value of the velocity ratio is known, the sectors can be constructed as follows: Let O and O', Fig. 145, be the fixed foci of a pair of equal and similar ellipses, sectors of which are to roll together with an angular velocity ratio which shall vary between the limits $\frac{3}{2}$ and $\frac{1}{2}$ while A, the driver, turns through an angle of 60 degrees.

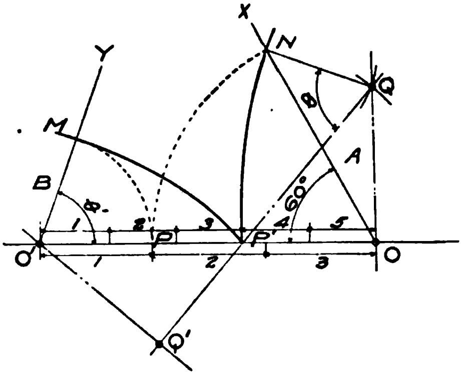


Fig. 145.

Divide OO', the line of centers, into $3+2$ equal parts and locate P. Divide OO' again into $2+1$ equal parts and locate P'. Lay off OX making an angle of 60 degrees with OO'. With O as a center and with radius OP, cut OX at N. With N as a center and radius $O'P$ draw an indefinite arc to the right of N and on P' as a center with radius $O'P'$ intersect the indefinite arc at Q. This point will be the other focus of the right-hand ellipse and QN and QP'

will be the contact radii of the left-hand ellipse. Lay off $\theta' = \theta$ and with radius $O'P = QN$ intersect $O'Y$ at M. The contact radii at the beginning of the motion will be OP' and $O'P'$ and at the end of the motion they will be ON and $O'M$. Draw the elliptical curves $P'N$ and $P'M$.

To find the other focus of the left-hand ellipse produce QP' and lay off $QQ' = OO'$. Ellipses constructed on OQ and $O'Q'$ are equal and similar and $P'QN$ and $P'O'M$ are similar sectors.

154. Logarithmic Spiral.—The logarithmic spiral is a plane curve generated by a point moving along a right line while at the same time the line revolves about one of its points as a center. The center is called the pole of the spiral and the right lines drawn from the pole to the curve are called radiants. It is a property of this curve that successive radiants, which include equal angles, form a series in geometrical progression. That is if equal angles 1, 2, 3, etc., be laid off about a center, the radiants which bound those angles will have values similar to the series 1, 2, 4, 8, etc. It follows from this that any radiant which bisects the angle between two other radiants is a mean proportional between them. This property enables us to construct a spiral arc when any two of its radiants are known.

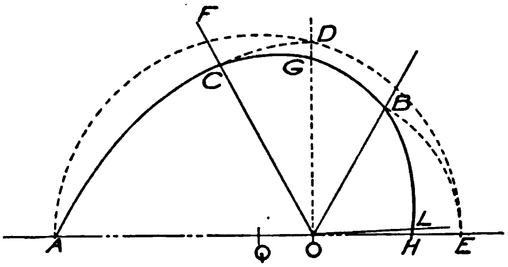


FIG. 146.

In Fig. 146, let it be required to construct the section of a logarithmic spiral having the pole O and passing through the points A and B.

Draw OA and OB and produce OA in the direction of E. Draw OF bisecting the angle AOB. Lay off $OE = OB$ and bisect AE at Q. With Q as a center draw a semicircle on AE and erect a perpendicular at O. By geometry OD is a mean proportional between OA and OE or its equal OB. Lay off $OC=OD$ and draw the curve through AB and C.

If a point is required to the right of OB, draw OL, making the angle $BOL=BOC$ and then make the distance OL a third proportional to OC and OB. Thus, $OC:OB=OB:OL$.

155. Laws of Contact; Logarithmic Spiral.—For any two non-circular wheels in external contact, the sum of the contact radiants equals the center distance. Logarithmic spirals may be constructed with the sum of their contact radiants equal to the center distance, in which case rotation about the poles will be opposite in direction. The spirals may also be constructed with the difference of their contact radiants equal to the center distance, in this case rotation will be the same in direction.

The velocity ratio, being inversely proportional to the contact radiants, will vary from a maximum to a minimum inversely as the radiants vary in length.

The equation of a logarithmic spiral is $r=a^\theta$.

Where r equals the length of the radiant,
θ equals the number of the successive angles counting from the origin to a given radiant,
a is any base of a system of logarithms.

Thus if $a=2$, the radiant which forms the side of the third successive angle of a spiral will equal 2 with the exponent 3, or $r=2^3=8$. In any spiral having the base a fixed in value, the spiral curve, and consequently its tangent, makes a constant angle with the radiants. Now a common tangent to two contact spirals must lie in the same straight line, hence every pair of radiants to the point of contact must lie in the same straight line. Therefore the point of contact must lie on the line of centers which is the condition of pure rolling.

The driving action will be positive in either direction as long as the contact radiants of the driver are increasing in length.

156. To Construct a Pair of Logarithmic Spirals.—To construct a pair of these curves for a given variation of the velocity ratio, we make use of the property that the sum of the radiants at the beginning of contact is equal to the sum of the radiants at the end of contact, in which case the curves are equal in length.

In Fig. 147 let O and O' be the poles of the spirals, let the spiral

A turn through an angle of 60 degrees and suppose the velocity ratio to vary from $OE : O'E = 3 : 2$ to $OF : O'F = 5 : 4$.

Divide OO' into $3+2$ equal parts and again into $4+5$ parts. Lay off the angle α and draw $O'X$. The contact radiants at the beginning of the motion are OE and $O'E$, and at the end of the motion they will be equal to OF and $O'F$. Lay off on $O'X$, $O'F'$ equal to $O'F$. Construct a spiral through the points F' and E by the method already explained. The curve constructed about the pole O will be a part of the same spiral and equal to it in length. It will turn through some angle β which we have to find, while the spiral *A* is turning through 60 degrees. With O' as a center and radius OF cut the given curve at F_2, with radius OE intersect the continuation of the same curve at E'. Draw the radiants $O'F_2$ and $O'E'$. The curve F_2E' is equal to EF' and the angle $F_2O'E'$ is the angle sought.

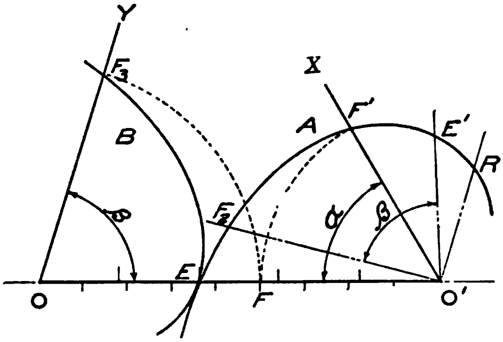


Fig. 147.

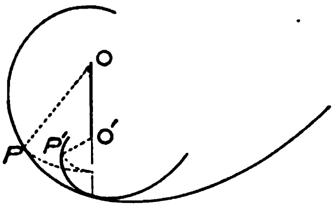


Fig. 148.

Lay off at O the angle EOY and on OY lay off OF_3 equal to $O'F_2$. Find points in the curve EF_3 by the method already described.

Fig. 148 shows a pair of spiral arcs in internal contact. With one spiral given the other may be constructed by the method just described, remembering that the difference instead of the sum of the contact radiants is equal to the center distance OO'.

157. Lobed Wheels.—The non-circular wheels thus far described will give one maximum and one minimum value of the velocity ratio for each cycle of motion. Lobed wheels with outlines derived from elliptical and logarithmic curves may be so constructed as to give more than one maximum and minimum value of the velocity ratio for each cycle. Such wheels will drive each other by pure rolling contact. The direction of rotation will be constant but the driving action will be positive during the part of the rotation only in which the radius of the driver is increasing in length.

158. Elliptical Lobed Wheels, Similar Pairs.—Two equal and similar ellipses are equivalent to a pair of unilobes having one maximum and one minimum value of the velocity ratio for each revolution. From these unilobes a pair of bilobes may be derived which will have two maximum and minimum values of the velocity ratio for each revolution. The method of deriving these curves is shown by Fig. 149. Draw the outer circles of reference and then divide the angular space *ABC* on one side of the major axis of the ellipse into any convenient number of parts. Draw the radii vectores *O I*, *O II*, *O III*, cutting the ellipse in points 1, 2, 3. Next, divide a quadrant of the ellipse, *AD*, into the same number of parts and draw the radii vectores *Ox*, *Oy*, *Oz*, etc. With center at *O* and with radius *O*1, intersect *Ox* at 1′. In a similar way intersect *Oy* at 2′, and *Oz* at 3′. A curve drawn through the points so found will be one-fourth of the outline of a

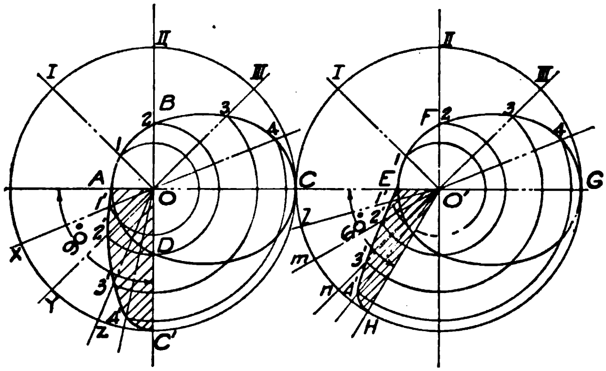


FIG. 149.

bilobe which will act in contact with a similar bilobe. The complete wheel is shown at Fig. 150 (*a*).

In the right-hand ellipse, if the angle *EO′H* be taken equal to one-third of *EFG* and the curve 1′, 2′, 3′ be found in the manner just described, then this curve will form one-sixth of the outline of a trilobe. This curve is then laid out symmetrically with respect to three axes to form the completed wheel as shown in Fig. 150 (*b*). This wheel will mate with a similar trilobe and give three maximum and minimum values of the velocity ratio for each revolution.

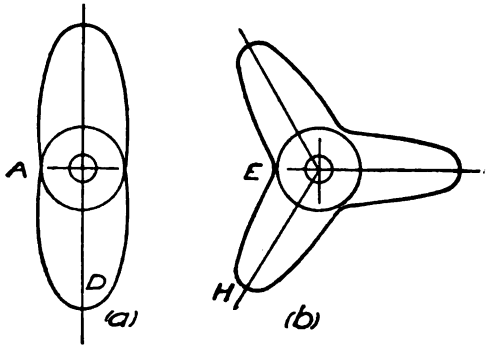


FIG. 150.

If we reduce the size of the angle *EO′H*, being careful to keep it a commensurate part of *EFG*, wheels with any number of lobes, within the limits of mechanical possibility, may be made by this method.

159. Elliptical Lobed Wheels, Dissimilar Pairs.—The wheels described in Art. 158 will act together only in similar pairs, that is, bilobe with bilobe, trilobe with trilobe and so on. Lobed wheels, however, can be constructed which will act together in dissimilar pairs, as a bilobe with a trilobe or a quadrilobe. In this case a part instead of the whole ellipse is used to form the derived curve.

In Fig. 151, let *AB* and *AC* be two arcs of equal length on similar ellipses. They are so selected that the angles which they subtend at *F* and *F′*, the fixed focii, have a given ratio, as 3 to 2. A bilobe and a trilobe which will act together can be made by contracting the angle AB to 90 degrees and AC to 60 degrees and reconstructing the arcs by the method of Art. 158. The derived arc AL will form one-fourth of the outline of a bilobe and AK will form one-sixth of the outline of a trilobe. The ratio of the contracted angles and the number of lobes on the wheels is also 3 to 2.

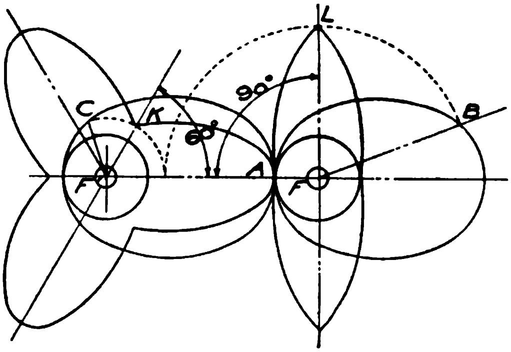


FIG. 151.

160. Interchangeable Lobed Wheels.—Fig. 152 shows a method of constructing elliptical lobed wheels that are completely interchangeable so that any wheel of the series will roll with any other. This is done by taking a series of ellipses having the same focal distance and having their minor axes in proportion to the numbers 1, 2, 3, 4. Then the first or primitive ellipse of the series will be a unilobe, the next larger will form the basis for the construction of a bilobe, the third will form the basis of a trilobe and so on.

The wheels are constucted from the basal ellipses by the method of contracting angles shown in Art. 158, Fig. 149. The laws of contact for these lobed wheels will be precisely like those for. the wheels already described.

161. Lobed Wheels from the Logarithmic Spiral.—In Fig. 153 (*a*), if the spiral DHC, subtending an angle of 180 degrees, be placed symmetrically on both sides of a straight line, it will form a unilobe. If the arc HC, which subtends an angle of 90 degrees, be placed symmetrically about two axes XX and YY, Fig. 153 (*c*), they will form the outline of a bilobe. Again if the arc which subtends an angle of 60 degrees at the pole of the same spiral be arranged symmetrically about three axes it will form the outline of a trilobe. Similar pairs of these wheels will act on each other in pure rolling contact, that is, two unilobes, two bilobes or two trilobes will act together but dissimilar pairs, as bilobe and trilobe will not work together because the spiral arcs which make their outlines are of different length.

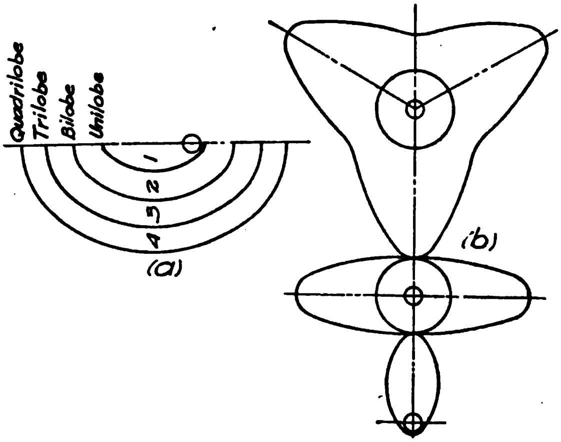


Fig. 152.

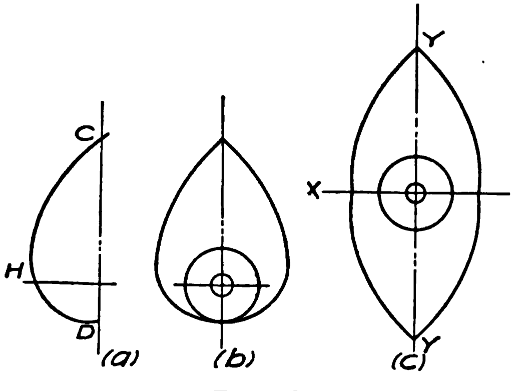


Fig. 153.

Lobed wheels from the logarithmic spiral may, however, be constructed by the following method, which will work together in dissimilar pairs:

Referring back to Fig. 146, suppose the spiral arc *GH* to subtend an angle of 90 degrees at the pole. In the manner already explained, this curve may be made to form one-fourth of the outline of a bilobe. Now if it is required to construct a trilobe from the same spiral which will roll in contact with this bilobe, we may lay off at *O* an angle of 60 degrees in such a position that the spiral arc intercepted between its radiants is equal in length to the arc *GH*. If *AOC* be the required angle, then *AC* will be equal in length to *GH* and $OA+OH$ will be equal to $OC+OG$.

PROBLEMS

1. Two cylindrical wheels are 6 and 9 in. radius and the larger wheel makes 20 r.p.m. Show that their angular velocity ratio varies inversely as their diameters and radii and directly as their revolutions per unit of time.

2. A cylindrical iron wheel is driven by a wooden one by direct contact. If the tangential force necessary to turn the iron wheel is 25 lb., and the coefficient of friction for metal on wood is 0.30, what is the required normal pressure between the wheels?

3. Lay out a pair of conical friction wheels to transmit a velocity ratio of 6 to 4, diameter of base of larger cone to be 5 in. and the angle between the axes 60 degrees.

4. Design a pair of elliptical friction wheels to transmit a velocity ratio varying between the limits $\frac{1}{5}$ and 5. Major axes to be 5 in. long.

5. Construct a pair of elliptical sectors which will transmit a velocity ratio varying between the limits $\frac{3}{5}$ and $\frac{4}{3}$. Angle of one of the sectors to be 75 degrees. Fixed focii 4 in. apart.

6. Construct a pair of logarithmic spirals which will transmit a velocity ratio varying between the limits $\frac{4}{3}$ and $\frac{2}{3}$. Angle subtended by one of the spirals 60 degrees. Distance between the poles 5 in.

7. Prove that the pressure at the bearings for a given load is less with grooved than with cylindrical friction wheels.

8. The focal distance of an ellipse is 3 in., major axis 5 in. Show how to derive from it a bilobe and a trilobe that will work together in rolling contact.

9. A non-circular segment of 60 degrees has radii at beginnning and end of contact of 2 and 4 in. Find the outline of a mating segment which has an initial radius of 2½ in.

CHAPTER IX

ROLLING AND SLIDING CONTACT

SCREWS, CAMS AND SLOTTED BARS

167. Screws.—In Fig. 154, if the plate A, having the inclined edge CD, be moved in the direction shown by the arrow, it will make contact with a follower B, along CD and B will be moved to B' by sliding contact. Now if this plate be wrapped around a cylinder whose axis is perpendicular to CE, then CD will form a helix and if the cylinder be rotated on its axis, the follower will be moved precisely as before. This piece of mechanism is called a screw. The follower is generally called a nut.

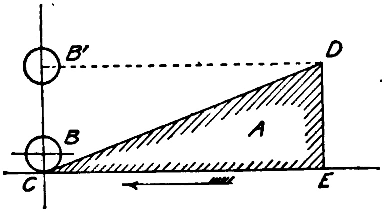


Fig. 154.

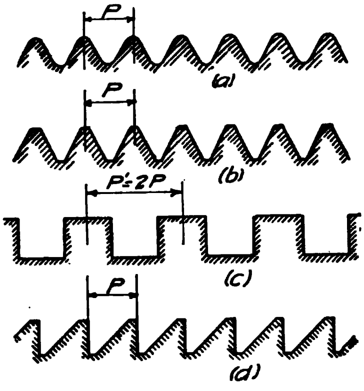


Fig. 155.

Screws are made by cutting a helical groove on a cylinder. The projection between the grooves is called a thread the form of which may be either square, V-shaped or buttressed. To prevent their sharp edges from injury, V threads are modified as shown by Fig. 155, in which (*a*) is known as the Whitworth thread and (*b*) as the Sellers or United States Standard.

The thread shown at (*c*), made square in cross-section, is the most interesting kinematically being the form generally used for transmitting motion.

When a screw is subjected to pressure in one direction only the buttress thread shown at (*d*) may be used.

168. Pitch of Screws.—The pitch of a screw is the distance from one thread to a corresponding point on the next one. As the pitch is often a small fraction of an inch it is customary to designate the pitch of screws by the reciprocal of this fraction. Thus, a screw having a pitch equal to $\frac{1}{6}$ in. is called a six-pitch screw.

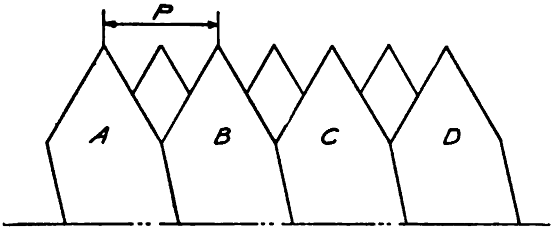


FIG. 156.

For convenience V threads have been standardized so that the pitch has a definite relation to the diameter of the screw; for example, a 1-in. standard screw always has eight threads to the inch. If the diameter of the screw is increased the pitch and size of the thread are both increased to correspond. The square threads are not standardized.

Screws may be made with either single or multiple threads. In Fig. 156, if *A*, *B*, *C*, are sections of a thread having a pitch *P*, then the follower will move forward a distance *P* for each revolution of the screw. This is a single thread. If *P* be bisected and the angles filled in as shown, each thread will have two V-shaped grooves, each one-half as large as the original, but the pitch distance P will remain unchanged. This is termed a double thread. By the same method multiple threads may be produced with any practical number of points to each pitch distance. The object of the device is to combine the advantages of a fine thread with a coarse pitch.

169. Right- and Left-hand Threads.—A screw is right hand if, when viewed from the end, the threads slope forward to the right as in Fig. 157 (*a*), and left hand if, when viewed from the end, they slope forward to the left as at (*b*). If a right-hand screw be rotated clockwise its follower will move toward the eye of the operator, and if the follower be rotated clockwise it will move away from the eye. The relative movements will be reversed if the screw is left hand.

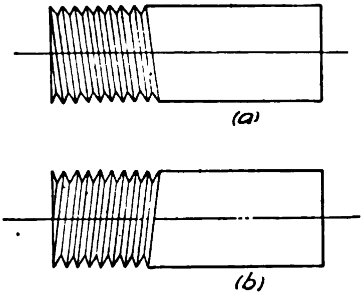


Fig. 157.

Threads are generally made right hand unless there is some apparent reason for making them otherwise. It is a convenient practical rule to remember that a nut should be turned clockwise when it is to be screwed on and counter-clockwise when it is to be turned off. If the thread is left-hand then the reverse will be the case.

170. The Propeller Screw.—In Fig. 158 is shown a form of the screw principle that is used for marine and aerial propellers and for the rotating elements of fans, exhausters and pressure blowers.

If a single-pitch screw be cut by two planes perpendicular to the axis and a short distance apart, the portion of the thread so cut off is called a blade. If the screw be multiple pitch there will be two or more blades on the part cut off. If such a screw be rotated in water or air it will cause flow in the medium, of a column equal in diameter to the diameter of the screw. If the screw be fixed to a ship the reaction of the column will impel the ship in the opposite direction. The speed of the column is equal to the pitch of the screw multiplied by the number of revolutions. The speed of the ship, if moving in water, will equal the speed of the projected column minus an imperical factor which is called the "slip," which may be from 5 to 10 per cent. of the speed of the column. The speed of air ships is influenced by air currents and other factors so that their propellers behave somewhat differently than those which rotate in water.

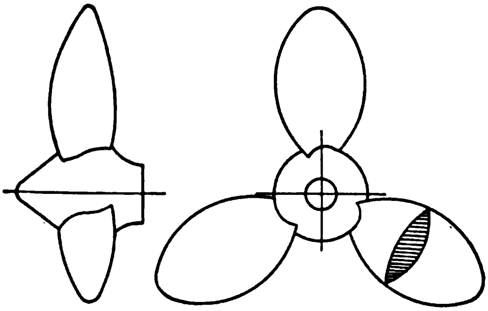


FIG. 158.

The diameter of a screw propeller is regulated by the amount of work to be done. The number of blades is generally four for large marine propellers and the efficiency tends to decrease if this number is exceeded. Fans and blowers have a greater number of blades, sometimes as high as six or eight. Air-ship propellers are made with but two blades, the desirability of this number being based on experimental results.

171. The Differential Screw.—In Fig. 159 let the screw AA' have threads of different pitch as N and M. Let the nut L be fixed so that it has no absolute motion either of rotation or translation, while K is so fixed that it can move along AA' but cannot rotate.

In one complete rotation the screw will enter the nut L by the pitch distance N while at the same time it will enter the nut K the pitch distance M. If the threads are both the same hand the resultant relative movement of *L* and *K* will be the algebraic difference of *M* and *N*. Thus, if the screw be rotated clockwise and *M* be greater than *N*, then *L* and *K* will separate by the distance *M–N*. If *N* be greater than *M*, *L* and *K* will approach each other by the distance *N–M*. If the screw be rotated counter-clockwise the relative movement of the nuts will be reversed.

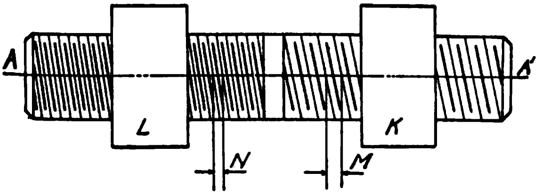


Fig. 159.

If the thread at *N* be right hand and that at *M* be left hand, *L* and *K* will approach each other a distance equal to $N+M$ when the screw is rotated clockwise and they will recede from each other the same amount when the screw is rotated counter-clockwise.

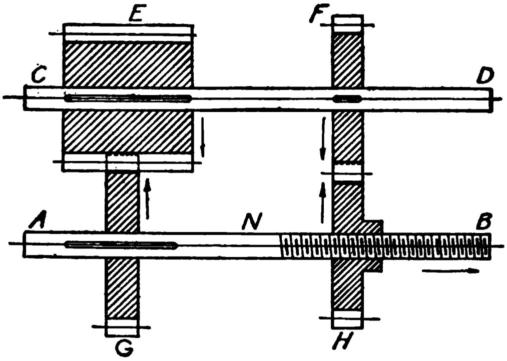


Fig. 160.

Fig. 160 is an application of the differential screw sometimes used on drilling machines to feed the drill forward into the work. *AB* is the follower spindle which carries the drill and *CD* is the driver. *E* and *F* are two pinions fixed rigidly on *CD*.

The pinion *H* is threaded to fit the screw *BN* and so fixed to the frame of the machine that it can rotate but cannot move in a direction parallel to the axis of *CD*. *G* is rigidly fixed to the spindle *AB*.

If *CD* be rotated as shown, it will drive the drill spindle, through pinions *E* and *G*, in the direction shown by the arrows. At the same time, pinion *F* will drive *H* and since *H* cannot move in the direction of its axis, it causes the screw and spindle to move toward the work in the direction of the arrow. Unless the thread on *NB* is very fine, the advance of the spindle due to *H* alone will be too rapid, but the rotation of the spindle which is due to *E* and *G* causes it to advance through *H* in a direction opposite to that caused by the rotation of *H* itself, hence its actual advance is the difference of the two motions. For example, if the pitch of the thread be 8 and the velocity ratio of the pinions is such that *G* makes two revolutions while *H* makes three, then in two revolutions of the spindle it will advance $\frac{1}{8}$ in. or $\frac{1}{16}$ in. per revolution.

The teeth of *E* are made long to allow *G* to slide forward without getting out of contact.

172. Cams.—A cam is a device for producing a variable, intermittent or reciprocating motion. Small cams are sometimes called wipers or tappets.

The mechanism has two elements designated as the cam proper and the follower.

The directional relation and velocity ratio are not comparable as in other forms of direct contact mechanisms, the relative motion of the two elements being of a wholly different nature.

The contact may be either pure rolling or pure sliding or a mixture of both.

The cam proper is generally made the driving element and the motion is positive for the working part of the follower's motion, although the latter may be made to return by gravity or by a spring.

The cam is one of the handiest devices available to the designer; it is made in a great variety of forms and is the basis of most of the peculiar and eccentric motions to be found in automatic machinery.

Cams may be studied according to the following classification:

Form of cam	Kind of follower	Path of follower	Motion transmitted
Plate	Roller	Straight	
		Curved	Continuous.
		Radial	
Cylindrical		Eccentric	Intermittent.
Conical	Bar	Plane	Uniform.
		Circular	
Spherical		Curvilinear	Accelerated.

173. Form of the Cam.—The cam proper is generally some modification of one of the following forms:

(1) The *face or plate* cam, Fig. 161 (*a*). As the cam revolves the follower is driven outwardly from the axis by making contact with the edge of the plate or the follower may be moved by a groove cut in the face of the plate.

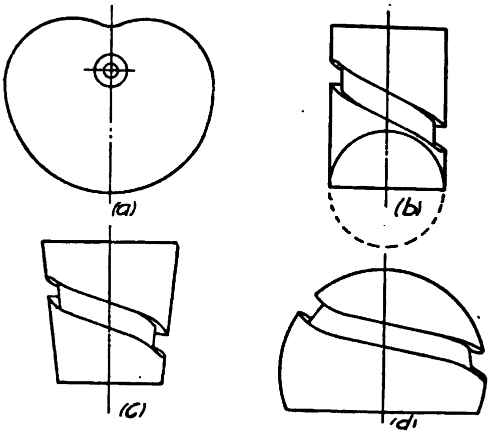


FIG. 161.

(2) The *cylindrical* cam, Fig. 161 (*b*). The follower is moved by a groove cut in the surface of a cylinder, the motion of the follower being in a plane parallel to the cylinder axis.

(3) The *conical* cam, Fig. 161 (*c*). The follower is driven by a groove cut in the surface of a cone, the motion of the follower being in a plane parallel to the elements of the cone.

(4) The *spherical* or *globe* cam, Fig. 161 (*d*). The follower is

driven by a groove cut in the surface of a sphere, the motion of the follower being spherical.

The curves which form the sides of the grooves or plate may be irregular or they may be regular curves as in the involute and helical cam.

The cams (*b*), (*c*) and (*d*) as shown, are called positive motion cams, that is, their follower will be displaced and returned to the starting-point wholly by the action of the cam, while (*a*) as shown, will displace the follower but will depend upon gravity or a spring to return it Cam (*a*) may be made a positive motion cam by cutting a groove in its face parallel to its edge or the same result may be obtained by making it in the form of two leaves, one extending upward from the center and the other downward, each leaf acting on separate but rigidly connected points of the fellower. This last form is known as the "main and return cam;" it is often used for heavy work.

174. The Follower and its Path.—The motion that the follower is required to have is the known element in the design of a cam mechanism. The form of the follower and the form of the cam which drives it depend upon the required motion.

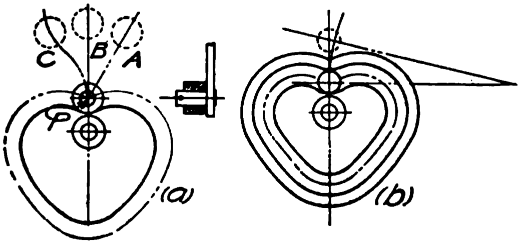


FIG. 162.

A roll follower is shown in Fig. 162. The roller is mounted on a hardened steel pin upon which it turns freely. The surface of the roller makes contact with the edge of the cam or with a groove cut in its urface. The path of the center of the pin is the path of the follower; the pin or the piece to which it is connected is, in fact, the real follower, the roller being used only to make better contact.

By arranging suitable guides the path of the follower may be made straight or curved or it may be a line either radial or eccentric with the center of the cam. Again, one end of the follower may be fixed so that the path will be the arc of a circle shown at Fig. 162 (*b*). This is called an oscillating follower.

Fig. 163 shows a bar follower. The edge of this follower makes contact with the edge of the cam. The bar may be either straight or curved and it may be guided so as to have rectilinear translation, curvilinear translation or be centered at one end so as to have circular motion.

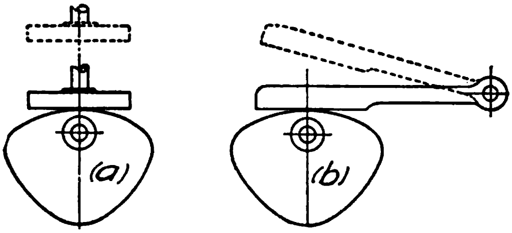


FIG. 163.

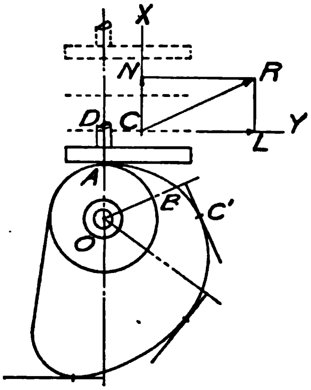


FIG. 164.

175. Contact of Followers.—If the roller be left off, Fig. 162, and the cam put in contact with the pin, the contact will be pure sliding; with the roller in place it will be pure rolling, because the point of contact will always be located in the line connecting the center of the cam and the center of the roller and consecutive points on one will come in contact with consecutive points on the other.

The contact of a bar follower is mixed rolling and sliding, for the point of contact always lies on the common normal to the contact surfaces and since for pure rolling, the point of contact must be located on the line of centers; if the contact is not mixed it will remain constantly on both lines. That the point of contact does not remain simultaneously on both of these lines is shown by Fig. 164.

The cam, in contact with the bar at *A*, turns through some angle *AOB* so that *OB* takes the position originally occupied by *OA*. During this motion the follower will be pushed upward to *D* and the common normal, carrying with it the point of contact, will move to the right to some position *C′*, and off of the line of centers.

If *CR* be laid off to represent the direction and motion of the point of contact at the instant when contact is at *C′*, and *CX* and *CY* are the common normal and common tangent, then with the aid of the parallelogram of motions, the amount of the rolling motion is found to be *CN* and the extent of the sliding to be *CL*.

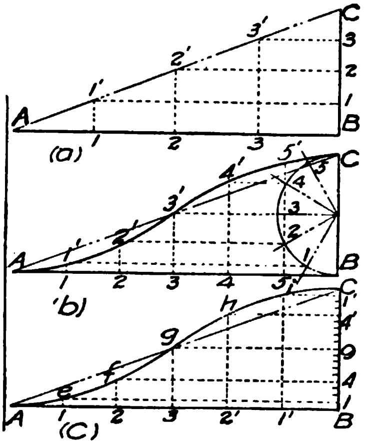


FIG. 165.

176. Motion Transmitted by Cams.—Followers are generally designed so as to have some form of reciprocating motion. They may be arranged to move continuously or their motion may be intermittent. In the latter case if the follower comes to rest while the cam continues to move, it is said to have a "dwell."

A follower may be made to move with uniform motion or it may have accelerated motion. When a follower has uniform motion it is constrained to move over equal spaces in equal intervals of time. If the motion is accelerated it may be accelerated uniformly or the acceleration may be varied. Fig 165 illustrates the principle of uniform motion and two forms of accelerated motion.

In sketch (*a*) let *AB* represent the linear motion of the cam and *BC* that of its follower. Divide *AB* and *BC* into the same number of parts and draw parallels and perpendiculars through the points of division. These lines will intersect in the points 1′, 2′, 3′ and the slope of a line drawn through these points will represent the relative velocity of the follower at different stages of its motion. Since *AC* is a straight line, the motion is uniform and the follower starts and stops at a condition of maximum velocity.

In sketch (*b*), *BC* is made the diameter of a semicircle which is divided into the same number of equal parts as *AB*. Parallels are again drawn intersecting in points 1′, 2′, 3′, 4′ and 5′. A line drawn from *A* to *C* through these points indicates by its slope that the motion is relatively slow at the beginning and end and at its maximum in the middle. The rate of acceleration is here a variable, being a maximum between the points 2 and 3. This is in fact simple harmonic motion.

If the motion be uniformly accelerated, the starting and stopping of the follower will be more gradual than in the simple harmonic motion just described. The acceleration curve is shown at Fig. 165 (*c*); it will be more flattened between the points *A* and *e* and between *i* and *C* than the previous curve. Its equation is of the form $y^2=Px$, a parabola.

Suppose a follower to start from rest and let

S = the space passed over by the follower in equal units of time,
a = the acceleration of the follower,
T = the number of equal units of time since the beginning of motion or the number of successive equal angles turned through by the cam since the beginning of motion.

Then:

$$S=\tfrac{1}{2}\,a\,T^2$$

The acceleration a may have any convenient value. Suppose it to be 2. Then substituting for a and T, we have for the first period of motion

$$S=1,$$

for the second period,

$$S=4,$$

for the third period,

$$S=9.$$

It will be seen that the distances that the follower has moved from the position of rest at successive instants, are in the series 1–4–9–16 . . . . . and that the distances passed over in each successive period form the series, 1–3–5–7–9 . . . . .
If 3 be taken as the fixed acceleration these series will become:

$$1\tfrac{1}{2}–6–13\tfrac{1}{2}–24\ .\ .\ .\ .\ .$$

and

$$1\tfrac{1}{2}–4\tfrac{1}{2}–7\tfrac{1}{2}–10\tfrac{1}{2}\ .\ .\ .\ .\ .\ \text{respectively}$$

The method of laying out this motion is shown in the sketch (*c*), for an acceleration of 2. BC and AB are first divided into two parts so that the motion will be first accelerated and then retarded. One-half of the time line AB is then divided into any convenient number of parts, say 3, and one-half of the space line is laid out in proportion to the series 1–4–9 . . . . . . Parallels and perpendiculars to AB are then drawn intersecting in points e, f and g; the curve is drawn through these points.

177. The Cam Curve.—The form of the cam curve which imparts motion to the follower is governed first by what the nature of that motion is to be and second by whether the follower is of the bar or roll type. Having determined the type of follower, the number of possible motions that it may have is unlimited, but the cam curve will always be some modification of the following:

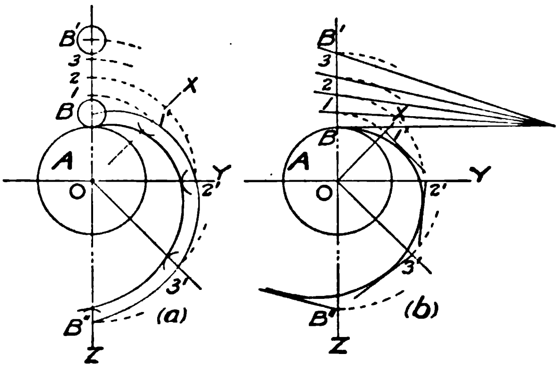


Fig. 166.

Let the plate A, Fig. 166 (*a*), drive the roller from B to B' while it rotates through some angle as 180 degrees. Divide BB' into any convenient number of parts at 1, 2, 3, etc., then divide the arc 180 degrees into the same number of parts, and draw the radii OX, OY, ... OZ. With center at O and radius equal to $O1$, $O2$, . . . OB' intersect OX at $1'$, OY at $2'$ . . . . . OZ at B''. A curve drawn through B, $1'$ $2'$ . . . . . B'', will be the locus of the center of the follower and the cam curve will be parallel to this line and at a distance inside of it equal to the radius of the follower.

In Fig. 166 (*b*), let the bar follower move from B to B' while the plate A rotates through a given angle. Find the intersections 1′, 2′, B″ as before and through these points draw lines which make the same angle with the radii OX, OY . . . . . OZ that the bar follower makes with BB' when the point 1′ is at 1, 2′ at 2, or B'' at B'. The outline of the cam will be a curve drawn tangent to the lines thus located.

178. Design of a Plate Cam with Roll Follower for Uniform Motion.—Suppose it is required to move the roll follower, Fig. 167, through the distance AB while the cam rotates through an angle of 180 degrees, and then allow it to return to its initial position while the cam is completing the balance of its rotation, the motion of both the cam and the follower to be uniform.

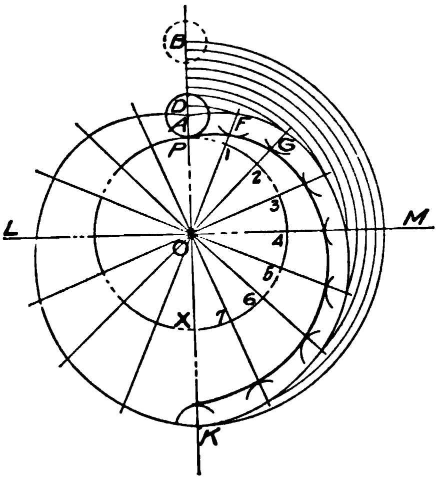


FIG. 167.

Take a radius OP, equal to the shortest radius of the desired cam and draw a base circle. Divide this base circle into any convenient number of parts and draw radial lines as shown, through $O1$, $O2$, ... OX. With a radius equal to OA, intersect $O1$ at F, with OD intersect $O2$ at G and so on until all of the radial lines around to X have been intersected, the last intersection, with radius OB, falling at K. Draw a curve through the points $A\ F\ G$ . . . . . K. This line will be the path of the center of the roller. With radius equal to that of the roller and centers at the intersections *F*, *G*, etc., set off toward *O* a part of the outline of the roller and then draw the cam outline tangent to the roller in its several positions.

The construction of the part of the cam to the left of the center line will be exactly like that just described.

179. Plate Cam, Bar Follower, Intermittent Motion.—Suppose the cam, Fig. 168, is required to drive a horizontal-bar follower through a distance *PC* while the cam turns through an angle *POH*, to hold the bar at rest while it turns through an angle *HOK*, and to allow its return to the initial position during the movement *KOP*.

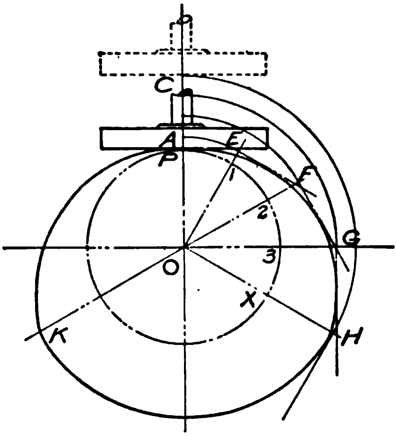


FIG. 168.

Draw the base circle with radius *OP* and divide the angle *POH* into any convenient number of parts and draw radii. Divide the path of the follower into the same number of parts proportional to those of *POH*. With radius *OA* intersect *O*1 at *E*. In like manner interest *O*2 at *F*, *O*3 at *G* and *OX* at *H*. Erect perpendiculars to the radii through these points and draw a curve tangent to them. While the cam revolves through the angle *POH* the follower will move through the distance $XH = PC$. The follower is to dwell at *C* while the cam turns through the angle *HOK*, *HK* is therefore an arc of a circle with radius *OH*.

During angular motion *KOP* the follower is returned by gravity or by a spring to its initial position, points on the curve *KP* being found as described for *PH*.

180. Cylindrical Cam, Roll Follower Hinged, Simple Harmonic Motion.—Fig. 169 is a cylindrical cam with vertical projection as shown. Let it be required to lay out upon this cylinder a cam groove which will drive a roll follower, hinged at *Q*, up and down through the angle α as the cylinder revolves.

Divide the semi-circumference about *O* into any convenient number of equal parts and draw parallel lines through the points 1′, 2′, 3′, ... , thus found. Lay off at *S* a semicircle having a diameter *TR*, equal to the travel of the center of the roller. Divide this semicircle into the same number of parts as the semicircle about *O* and draw through the points of division another set of parallel lines intersecting the first set in the points *A*, *B*, *C*, *D*, *E*. Then draw the center line of the groove through these points and the points *R* and *T*′.

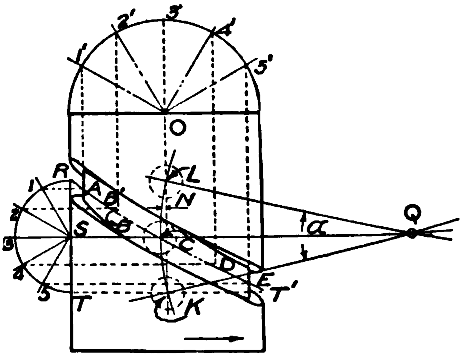


FIG. 169.

Since the center of the roller will move in the arc of a circle drawn from *Q*, points on the true path of its center must be offset from the intersections found, by the amount that this arc deviates from a straight line. Thus, the offset from the point *B* will be *BB*′, equal to *N*.

The sides of the groove will be drawn parallel to the center line, and at a distance from it equal to the radius of the roller.

The depth of the groove will depend upon the thickness of the roller.

This cam will drive the follower positively in both directions. With the motion of the cylinder in the direction shown by the arrow, the follower will move from *C* to *L* and return again to *L*, during the first half of a rotation and during the second half it will move from *C* to *K* and return.

181. Parabolic, or High-Speed Cam.—Let *O*, Fig. 170, be the center of a plate cam which is to drive a hinged bar follower upward, through an angle of 15 degrees, while the cam rotates through 180 degrees, follower to return to its initial position during the last half of the cam's rotation.

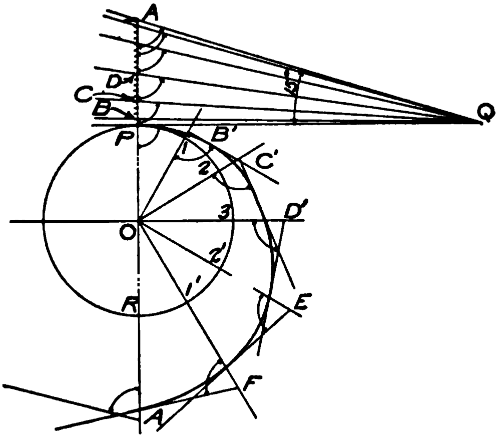


FIG. 170.

Divide the distance *PA* into two equal parts, also divide the semicircle about *O* into two equal arcs. Sub-divide each of these arcs into any convenient number of parts, as three, and draw the radii *O*1, *O*2 and *O*3. Divide *PD*, the half travel of the follower, into the square of the number of divisions on the quadrant *P*3. Then with radius $OB = (OP + 1^2)$ cut *O*1 at *B'*. With radius *OC* equal to $(OP + 2^2)$ cut *O*2 at *C'*. With radius *OD* equal to $(OP + 3^2)$ cut *O*3 at *D'*. Divide *DA* also into nine equal parts and with radius *OA* cut *OR* at *A'*, with radius $[OA - 1^2]$ cut *O*1' at *F'*, with radius $[OA - 2^2]$ cut *O*2' at *E*. Through the point

B′ draw a line making the same angle with the radius that the follower in the position *QB* makes with *OA*. Through *C′* draw similarly, a line making the angle *OCQ* with the radius, and so on for all the points. Draw a curve tangent to all these lines. Then the follower will move upward with uniformly accelerated motion to *D* and with uniformly retarded motion from *D* to *A*. The other half of the cam will be constructed in exactly the same way and the follower will start downward with uniformly accelerated motion and gradually come to its starting point with uniformly retarded motion.

182. The Yoke Cam.—Fig. 171 represents a cam having virtually two followers which always remain a fixed distance apart. The curves *AB* and *CD* are found by plotting the movements of the cam and follower by any of the methods described according to the form of the motion required.

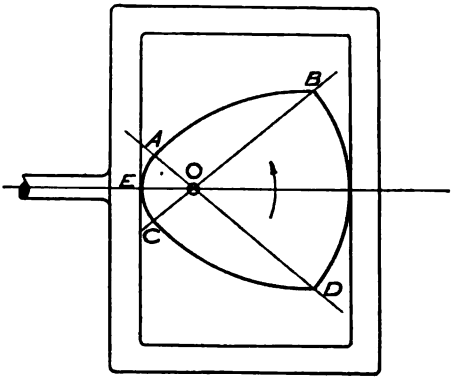


Fig. 171.

With the cam rotating as shown by the arrow, while contact is on the curve *BA*, the follower is pushed to the left by the distance *OB–OA*. While contact is between *B* and *D* the follower has a dwell, *BD* being the arc of a circle drawn from the center *O*. While contact is between *D* and *C*, the follower is pushed back to its initial position and dwells there while contact is between *C* and *A*, *CA* being also the arc of a circle drawn from the center *O*.

All straight lines which pass through the center *O* and cut the cam in two points are the same length hence the cam is always in contact with the follower in two points. The follower may be made in the from of a double bar as shown or it may be made with two rollers mounted a fixed distance apart. In the first case it is called a "constant breadth cam" and in the latter a "constant diameter cam."

This cam is sometines made with three circular arcs drawn from points D, B and E. It has been applied to driving certain kinds of valve gears.

183. Wheel Cams.—Cams are sometimes designed to drive a follower by means of a strip of metal fastened to the surface of a rotating wheel. This form of construction is frequent in automatic screw machinery. Fig. 172 (*a*) is a side view of a cam wheel, and (*b*) is the development of the same. These strips can be so formed as to give any desired form of motion to the follower. The principle is the same as that of a cylindrical cam, the difference being in the method of construction. The angle α ought not to exceed 30 degrees otherwise the friction becomes to great for satisfactory action.

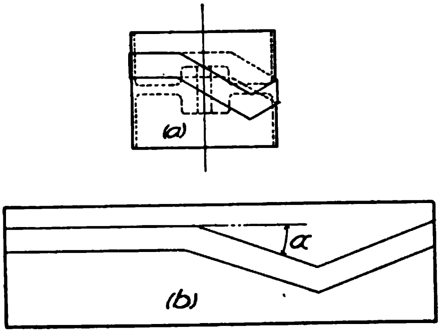


Fig. 172.

184. Cam Time Tables.—In designing automatic machinery where a large number of similar or successive operations are required, it is convenient to refer the time at which the different cams begin and end their movements, to a common starting-point. These movements can then be plotted in the form of curves as shown by Fig. 173. The length AB represents one complete cycle of operations of the machine and may mean one or more revolutions.

AC represents the maximum travel of any follower. The zero point for starting all followers is referred to the left-hand line of the chart, the actual starting-point for a given follower is denoted by some vertical line as *FG*, and the distance of this line from the origin represents the angle of advance of the corresponding cam.

Suppose a simple case of a machine with three cams which operate as follows. The first follower starts when machine is in 0 position, moves 3 in. while its cam moves 120 degrees, rests while cam moves 120 and returns to original position while cam moves last 120 degrees, simple harmonic motion. Second follower starts when first cam has completed 120 degrees, moves 2½ in. while cam moves 180 degrees and then returns to original position while cam revolves through last 180 degrees, motion parabolic. Third cam starts when machine has completed half of one revolution and drives the follower 4 in. in first half of its motion and returns it to original position during last half. Motion uniform.

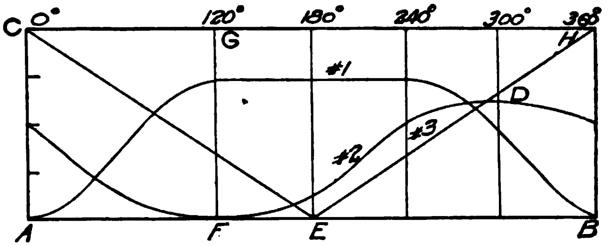


FIG. 173.

The chart for these cams would be laid out as shown. The curve for No. 1 begins at *A* and from 0 degrees to 120 degrees is a curve of simple harmonic motion, as already explained. From 120 degrees to 240 degrees the follower stands still, as represented by the straight line. From 240 degrees to 360 degrees the curve again represents simple harmonic motion. In No. 2 the parabolic curve starts at *F* and reaches its maximum height at *D* and then reverses to the starting-point. The distance *AF* represents the advance of this cam.

No. 3 starts at *H* and its motion is represented by two straight lines *CE* and *EH*.

184. Slider Blocks. Oldhams' Coupling.—Circular and elliptical motion may be transmitted by sliding contact by means of a block sliding in a groove, either element being the driver.

The Oldhams' coupling, Fig 174, transmits circular motion between axes which are parallel but not in the same straight line.

The coupling as shown is made up of three flat disks, two of which are rigidly fastened to the ends of the connected shafts, and having diametral grooves cut across their faces. The third disk has a projection or bar on each face, the two bars forming a cross with the arms at right angles. These bars fit the grooves of the other disks.

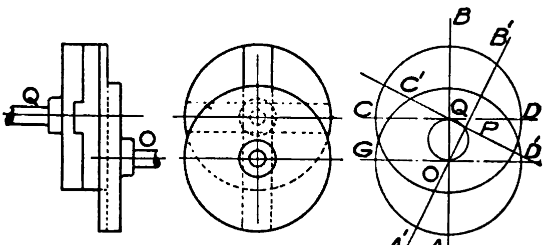


FIG. 174.

While the driving disk makes one revolution its bar and groove will slide on each other a distance equal to twice the distance between the axes and the following bar and groove will have a similar relative motion. In the diagram (*c*), the centers of the disks are at *O* and *Q*, and for the position of the coupling shown, the center lines of the bars are at *AB* and *CD*, crossing each other at *Q*. If the disk *Q*, be turned through any angle the center of the cross will move to some new position *P*. For every position of this center, we have the triangle *OPQ* in which the side *OQ* is constant and *OPQ* is a right angle. Therefore, by geometry, the locus of the center of the cross is the small circle whose diameter is equal to the distance between the axes.

While the center of the cross is moving from *Q* to *P* a point on the rim of the disk moves from *C* to some point *C′*. The center line of the bar for this new position is drawn through *C′* and *Q* and the center line of the follower disk is drawn through *O* and perpendicular to $C'D'$. Evidently the angle CQC' is equal to the angle AOA'; therefore the angular velocity of both disks is the same.

186. Elliptical Paths.—Let the grooves Fig. 175 be made to intersect at right angles and a bar PCD, be mounted on pins that slide freely in the grooves. Then if C be made to slide once across and back again in the horizontal groove, while D does the same in the vertical groove, the point P will move in an elliptical path. Because if PD be assumed equal to half the major axis of some ellipse and PC equal to half the minor axis of the same ellipse then from the figure;

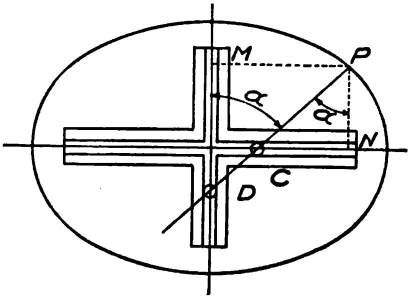


FIG. 175.

$$\frac{PM}{PD}=\sin\alpha$$

and

$$\frac{PN}{PC}=\text{Cos}\ \alpha$$

then by squaring and adding these terms

$$\left(\frac{PM}{PD}\right)^2+\left(\frac{PN}{PC}\right)^2=\sin^2\alpha+\text{Cos}^2\alpha=1$$

This is the equation of an ellipse when PM and PN are the coordinates of any point P.

The above principle is made use of in turning or boring elliptical pieces of machinery, the apparatus being shown in Fig. 176. A represents the head stock of a lathe, having fastened to the spindle S a face plate B. This face plate has two dovetailed grooves, XX' and YY' at right angles across its back.

The attachment between the spindle and the face plate is made by means of a block which slides in the groove XX'. The center of the plate is at O, the intersection of the center lines of the grooves. An annular ring R with center at D is fastened to the headstock by lugs which are bolted to the slotted lugs EE'. On the outside of this ring is a slip ring M having lugs which fit into the groove YY' on the back of the face plate. The action is as follows: The center of the spindle O corresponds to C of Fig. 175, and D, the center of the rings, corresponds to the point D of Fig. 175. The spindle S passes through the annular ring and drives the face plate by means of the sliding block in the groove XX'. The face plate acting on the lugs of the slip ring cause it to turn with an eccentric motion with respect to the center of S.

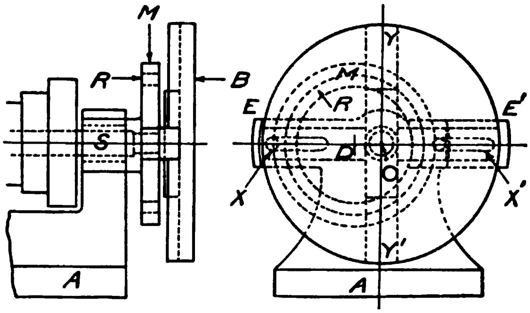


FIG. 176.

The result is, O and D being stationary, that the face plate has a reciprocating motion in two directions, one motion tending to slide the center line of groove XX' on point D, the other to slide YY' with respect to O. The resultant of these two motions gives an elliptical path to any point on the face plate.

PROBLEMS

1. An 8-pitch screw makes 24 r.p.m. What is the axial velocity of the nut?
2. Explain the difference between a single and double thread.
3. Suppose CD in Fig. 160 makes 30 r.p.m. and the velocity ratio $\frac{E}{G}=\frac{2}{3}$

while that of $\frac{E}{H} = \frac{10}{7}$. If N is a 16-pitch screw, what will be the direction and speed of the screw if CD rotates as shown.

4. Which way should the nuts on a wagon wheel be turned to screw them on, clockwise or the reverse? Explain.

5. Prove that the contact between a plate cam and a bar follower is mixed rolling and sliding.

6. Lay out a plate cam to drive a bar follower 2 in. vertically upward while the cam revolves 180 degrees. Motion uniform. Base circle of cam 4 in. in diameter.

7. Lay out a cylindrical cam 4 in. in diameter to drive a $\frac{1}{2}$ in. roll follower in a line parallel to the cam axis, a distance of 3 in. while the cam makes a complete revolution. Motion of follower to be simple harmonic motion.

8. Lay out a plate cam that will drive a hinged roll follower 1 inch in diameter positively in either direction, through an angle of 30 degrees. Length of roller arm 6 in. Roll advances while cam moves 120 degrees, rests for 120 degrees, returns during last 120 degrees of motion. Cam to be suitable for a high speed.

9. A yoke cam is required to draw the wedges beneath the seats of a railroad drawbridge. The throw of the cam is to be 12 in. and its motion uniform. The operation of opening the bridge is as follows: The total time required to turn the bridge from drawing to reseating the wedges is six minutes. The rail locks are first operated, requiring fifteen seconds, then the wedges are drawn in thirty seconds. The follower is required to dwell while the rail locks are being operated and the cam to rest while the bridge is turning. Lay out the cam.

10. Lay out a time table for the bridge above, the cam to stop every time in such a position that the wedges will start as soon as the cam begins to rotate.

CHAPTER X

DIRECT CONTACT. ROLLING AND SLIDING MOTION

SPUR GEARS

187. A Gear is a wheel having on its rim projections which are called cogs or *teeth.* The teeth are so made as to interlock with similar teeth on a mating wheel and transmit motion from one to the other. In action, the sides of the teeth on one of the wheels press against those of the other with a rolling and sliding contact and the relative motion of the two wheels is precisely like that of the friction wheels shown in Fig. 127, Art. 138.

A pinion is a gear having a small number of teeth.

188. Classification of Gears.—There are so many different kinds of gears that an accurate and complete classification is nearly impossible. They may be classified according to the shape of the wheels or according to the form of the teeth, or they may be classified as in this chapter according to the position of the axes. These classes are subdivided as follows:

Axes parallel.	Spur gears.	Pitch surfaces cylinders.
	Annular gears.	
	Elliptical gears.	
	Chain gears.	
	Pin gears.	
	Twisted gears.	
	Irregular gears.	Pitch surfaces irregular.
Axes at an angle in same plane.	Bevel gears.	Pitch surfaces cones.
	Elliptical gears.	
	Face gears.	Pitch surfaces cylinders.
Axes in parallel planes.	Helical gears.	
	Worm gears.	
	Skew gears.	Pitch surfaces hyperboloids.

189. Spur Gears.—A spur gear is a cylindrical wheel having teeth placed radially around the rim, the elements of the teeth being parallel to the axis of the wheel, Fig. 177.

If the whole of the tooth is made to project beyond the rim as shown at Fig. 178 (*a*), the mating gear will have notches cut inside the rim in which the teeth of the first gear can act. This is the simplest form of the spur gear but it is open to the objection that only one of the gears can be used as the driver and contact between the teeth takes place only on one side of the vertical center line. The usual form of construction is shown at Fig. 178 (*b*) where both gears of a pair are made with projections around the rims and spaces inside the rims into which the opposing teeth act. These gears are interchangeable and contact takes place on both sides of the vertical center line.

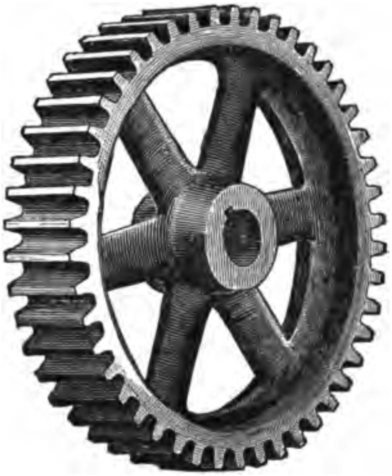


Fig. 177.

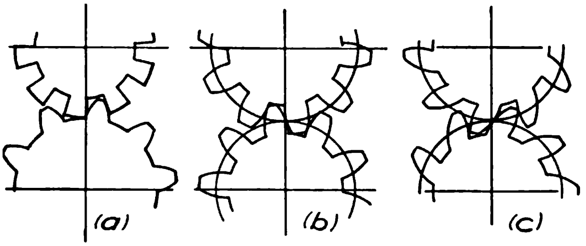


Fig. 178.

Gears that are required to run in one direction only may be made as shown at Fig. 178 (*c*). This is called the buttressed tooth. The form is suitable where a great pressure is put upon the tooth and always in the same direction.

Spur gears are the simplest of all the gear forms and the easiest to make. When carefully constructed they run quietly and with good efficiency but if badly made they cause trouble from friction, noise and lost motion.

190. Tooth Curves.—Spur gear teeth are studied on a plane of reference which is taken perpendicular to the axes of the gear. The outlines of the gear teeth as seen on this reference plane are curves which depend for their form on the prescribed line of action. The line of action being the locus of the point of contact between the sections of a pair of teeth.

Spur tooth curves are formed geometrically by rolling a describing line or *directrix* on another line called a *director*. In order that two mating gears may have the correct relative motion these rolled curves must have the following properties:

Any pair of tooth curves which act in contact must have the same directrix and this directrix will be the locus of the point of contact between the tooth curves, i.e., their line of action.

The common normals to a pair of tooth curves must be arranged in consecutive order without crossing each other and they must all meet in a common point on the line connecting the centers of the pitch circles. This is called the pitch point.

An indefinite number of pairs of curves can be found which will fulfil these conditions but only two regular curves the *involute* and the *cycloidal* have been used to any extent for forming spur tooth outlines.

191. Influence of the Line of Action.—Fig. 179 shows the relation between various tooth curves and their lines of action. The tooth sections shown are those of a rack, and if the latter be regarded as a gear having an infinite number of teeth, whatever is true of these sections is true of every gear in the system.

If a straight line be taken for the side of the rack tooth as at (*a*) the line of action will be a straight line perpendicular to the side of the tooth and all teeth having a straight line of action belong to the *involute* system.

At (*b*) cycloidal curves are taken for the outlines of the rack tooth, the lines of action are circles and all gear teeth having circles for their lines of action belong to the *cycloidal* system.

If arcs of circles be selected as the outlines of the rack tooth the lines of action will form a loop, part of which is shown at (*c*). This loop is the conchoid and the teeth are *segmental.*

At (*d*) a section of a parabola is taken for the rack tooth, the line of action is a double loop or "hour glass" curve and the teeth are defined as *parabolic.*

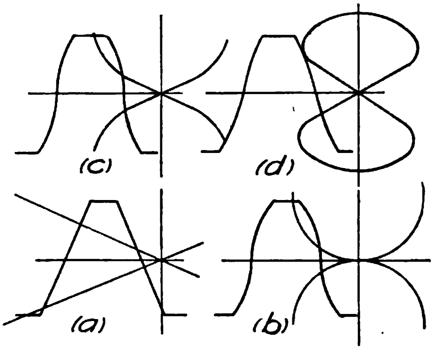


FIG. 179.

192. Elements of a Spur Gear.—Let Fig. 180 represent the section of a pinion on a plane perpendicular to the axis. The pinion is shown in contact with a gear on one side and a rack on the other. *AB* is called the pitch circle or the "pitch line."

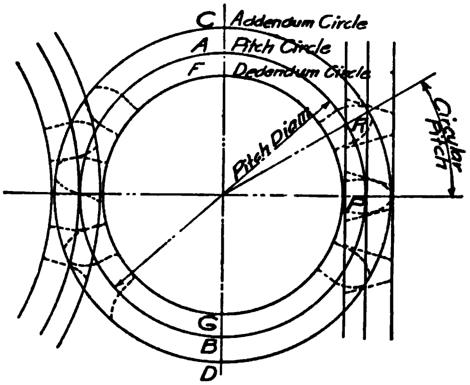


FIG. 180.

This circle is tangent to the pitch circle of the gear and to the pitch line of the rack and corresponds to the periphery of a friction wheel. The teeth of the pinion extend within the pitch lines of both the gear and the rack and these in turn extend within the pitch circle of the pinion.

The diameter of a gear is understood to be the diameter of the pitch circle.

The outer ends of the teeth are limited by the addendum circle and the diameter of this circle is called the outside diameter of the gear. The circle at the base of the teeth is called the dedendum circle.

The circular pitch is the distance *PR*, measured on the pitch line, from one tooth to a corresponding point on the next one. This distance is the same in pinion gear and rack.

193. Parts of a Gear Tooth—(Fig. 181.) The *addendum* of a gear tooth is the part that lies outside of the pitch circle.

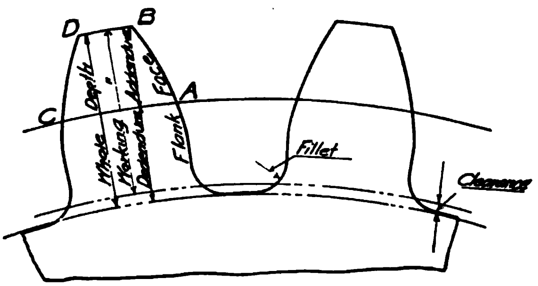


FIG. 181.

The *dedendum* is the part of a tooth between the pitch circle and the root circle.

The *face* of a tooth is the curved surface between the pitch line and addendum line, Fig. 181, *AB*. *CD* is the back face.

The *flank* is the curved surface between the pitch and dedendum lines.

Clearance is the space between the addendum circle of one gear and the dedendum or root circle on its mate.

Backlash is the difference between the width of a tooth and space. It is necessary in cast gears because of irregularities. It causes lost motion when gears are reversed in direction and ought not be necessary in cut gears.

The *root* of a tooth is the part which joins the rim of the gear. It should be rounded off with a small radius or fillet.

The *working depth* of a tooth is the addendum plus the dedendum. The whole depth is equal to the working depth plus the clearance.

194. Pitch, as used in the mechanical sense, means the distance between corresponding points in a series of cogs or teeth. Much confusion has resulted from using the same word to express three different ideas.

(a) *Circular Pitch.*—If the *circumference* of a pitch circle be divided into as many parts as there are teeth on the gear, the length of each part is the *circular pitch.* It is a definite distance usually expressed in inches or fractions thereof. For example, if the circumference of a pitch circle is 40 in. and the gear has twenty teeth, the cicular pitch or the distance from one tooth to the next, measured on the arc, is 2 in. and each arc of 2 in. will contain one tooth and one space.

(*b*) *Diameter Pitch or Module.*—If the *diameter* of a pitch circle be divided into as many parts as there are teeth on the gear, each part is a *diameter pitch or module.* The module is also a definite distance and bears the same relation to the diameter of the pitch circle as the circular pitch does to the circumference. Thus if a gear is 10 in. in diameter and has 20 teeth, the module is ½ in.

(c) *Diametial Pitch, or Pitch Number.*—If the number of teeth on a gear be divided by the number of inches in the pitch diameter, the quotient will be the *diametral pitch.* This is not a concrete number, it is the ratio of the number of teeth on a gear to the number of inches in its pitch diameter. Thus if a two-pitch gear has twenty teeth, it will be 10 in. in diameter, and conversely, if a gear having twenty teeth is 10 in, in diameter it must be a two pitch-gear. This diametral pitch is the reciprocal of the module.

The relation between the diametral pitch and the circular pitch is expressed by the formula,

$$PP' = \pi$$

where P is the circular and P' the diametral pitch.

The diametral pitch as used here is properly a pitch number, the module should be called the diametral pitch. The inversion of these terms in American practice is somewhat confusing but the definitions as given above accept the conditions as they exist.

195. Conjugate Teeth.—Any two curves, regardless of their form, may be used for tooth outlines provided they fulfil the conditions of Art. 190. Two teeth which will roll together properly are called conjugates. If the outline of one of a pair of conjugates is known the other may be found by the following process:

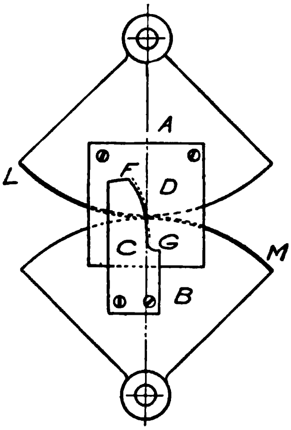


FIG. 182.

A and *B*, Fig. 182, are two boards whose edges are arcs of given pitch circles. Fasten to them a thin, flexible band, *LM*, so that they will roll in contact without slipping. Make a sheet metal templet, *C*, of the existing tooth and fasten it to *B* with its under surface raised a little above the boards, and having the pitch point of the tooth exactly over the edge. Fasten to *A* another piece of metal, *D*, in such a manner that it can slide underneath *C* as the boards rotate.

Hold the edges in contact and roll one on the other. Trace on *D* in a number of positions the tooth outline *C*. A curve *FG* which is tangent to all these successive positions will be the outline of the required tooth.

196. Designing and making Gear Teeth.—Two methods of forming a correct tooth outline are possible:

(a) Roll out the tooth curve using the line of action as directrix, make a templet to fit the rolled curve and shape the teeth to the templet.

(b) Select a rack tooth as explained in Art. 191 and use this rack tooth as a cutter to form the teeth of the gear.

Gear teeth are made by:

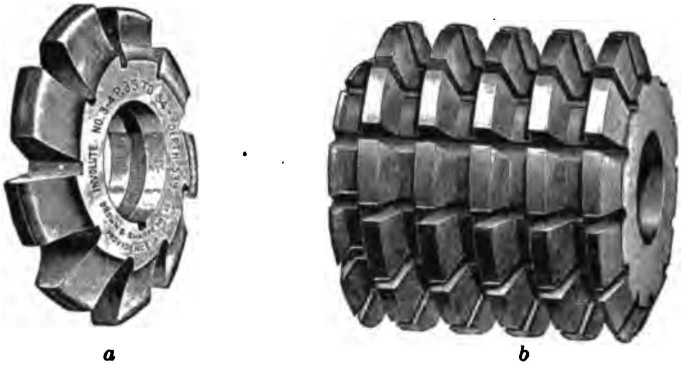
(1) Casting.
(2) Milling.
(3) The linear planing process.

(4) The hobbing process.

(5) The moulding planing process.

Cast teeth are first laid out on paper by describing the tooth curves geometrically. From the drawing a templet is made, usually of wood, and this templet may then be used as a pattern to form the tooth by the ordinary sand moulding process.

Milled gears are made by cutting out the spaces between the teeth with a circular cutter. The cutter, Fig 183 (*a*), is first ground to the correct tooth shape which is found geometrically. Theoretically, a different cutter must be used for every different diameter and pitch of gear, but this would necessitate so many cutters, that those of a given pitch are generally used to cut gears of diameters that range between certain limits. Brown and Sharpe carry in stock eight involute cutters for each pitch, the first one can be used on anything from a rack to a gear of 135 teeth, and so on down to the eighth which will cut a gear of twelve or thirteen teeth. These cutters are not quite right for gears at the lower or upper limit but they are sufficiently accurate for practical purposes.



a *b*

FIG. 183.

By the planing process the teeth are formed by planing out the spaces between them with a tool which has a reciprocating motion. The tool is guided by a templet having an outline like that of the tooth being cut. The outline of the templet may be found as in the two previous cases.

In the hobbing process a circular cutter or hob, Fig. 183 (*b*), is first made with an axial cross-section like that of a rack of the system. Slots are milled lengthwise in this cutter to form cutting edges. Hob and gear blank are then mounted together on a machine so that both can be rotated on their axes and at the same time their axes brought gradually together. In this way the hob is made to generate the teeth, action between the gear and the hob being precisely like that of a gear and rack of the system.

If we imagine a pinion of hardened steel and a gear of some plastic material and if these be rotated at the proper speed ratio and at the same time their axes be brought gradually together, the pinion will form teeth, on the gear, of correct shape to mesh with it. The materials of a gear being too hard to be moulded in this way, a tool corresponding to the pinion is made with cutting edges and given at the same time a reciprocating and a rotary motion so that the material between the teeth is planed out. The action between the cutter and gear is precisely like that of the completed gear and pinion. This is known as the moulding planing process.

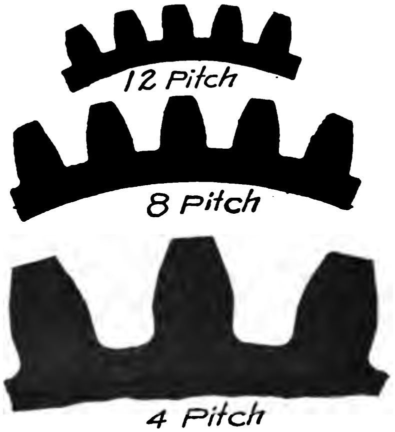


FIG. 184.

197. Standard Gear Teeth.—The pitch diameters of gears are determined by the distance between the axes to be connected and their relative speeds. The pitch of gears is determined by the load or force to be transmitted. The coarser the pitch is made the stronger the teeth will be, as shown by Fig. 184 where the teeth are drawn actual size. Formulas which give the relation between the pitch and the force to be transmitted can be found in the text-books on machine design. The following formulas give dimensions of standard gears.

Let P = diametral pitch.
P_c = circular pitch.
A = Addendum of tooth.
D = Dedendum of tooth.
T = Thickness of tooth.
N = Number of teeth.
D_p = Pitch diameter in inches.
D_o = Outside diameter in inches.
C = Clearance.
F = Fillet.
S = Circumference.

Then (a) $P_c = \frac{S}{N}$

(b) $P = \frac{N}{D_p}$

(c) $P = \frac{3.1416}{P_c}$

(d) $A = \frac{1}{P}$

(e) $C = \frac{1}{8}A$

(f) $D = A + C$

(g) $T = \frac{S}{2N}$

(h) $D_o = D_p + 2A$.

The radius of the fillet may be made equal to one-sixth or one-seventh of the space between the teeth, measured on the addendum line.

The amount of back lash depends upon the nature of the work. In rough gears it is sometimes equal to an eighth of an inch or even more.

198. Stub Teeth.—The standard tooth of Art. 197, was originally made of wood. To make the tooth contact continuous, long arcs of approach and recess were necessary because of the flexibility of the teeth and of the supports. The greater stiffness of metal gears and supports would indicate that the standard tooth is unnecessarily long, particularly as the tendency of long teeth is to decrease the efficiency of the gears: (a) By increasing the contact at the ends of the arc of approach and recess where the friction is greatest; (b) by throwing excessive pressure against the bearings due to the obliquity; (c) by increasing the number of teeth that are in contact at one time. The long teeth of the old standard are also harder and more expensive to make than teeth need to be.

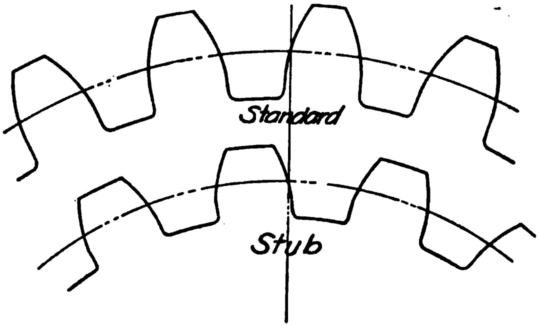


FIG. 185.

In spite of some obvious disadvantages the standard tooth has many advocates and the most suitable dimensions of a tooth for general purposes is an open question. Undoubtedly there should be but one standard so that all teeth of the same pitch would be interchangeable. The shortened or stub tooth is shown in comparison with the standard tooth in Fig. 185. It is made with obliquities of both 14½ and 20 degrees and with the following dimensions:

Addendum, $\frac{0.7854}{P}$

Dedendum, $\frac{0.9424}{P}$

Working depth, $\frac{1.5708}{P}$

Whole depth, $\frac{1.7278}{P}$

Clearance, $\frac{0.157}{P}$

where P is the diametral pitch.

199. Involute Teeth.—The simplest and in many respects the best form of gear tooth, and the one that is now most commonly used, is the involute.

Theoretically, the involute is the curve traced by the pole of a logarithmic spiral when the latter is rolled on the pitch circle of a gear. Since the logarithmic spiral has an infinite number of convolutions about its pole, the latter can never be made to reach the pitch circle exactly, hence the construction of the involute by this method is a practical impossibility, even an approximate construction being a laborious process. An involute may also be generated by a point in a straight line when the line is rolled on a circle. It happens to be a peculiar property of the involute that it will work satisfactorily as a tooth curve when generated by this method on a circle and (located within the pitch circle) called the base circle of the gear.

The larger the base circle, the nearer the involute approaches a straight line and for a circle of infinite radius it becomes a straight line. Hence the sides of involute rack teeth are straight and perpendicular to the pressure line of the mating gear.

In generating involute teeth automatically the cutter is made in the form of a rack, the sides of the rack teeth being straight they are easily and accurately produced. As the rack is a gear with an infinite number of teeth every gear generated will work with it and with each other.

The involute tooth has the advantages of simplicity of form and construction. It will work with another tooth when their pitch points are not accurately adjusted, also the pressure at the bearings of the gear is constant in direction. Its principal disadvantage is interference which sometimes makes it necessary to modify the tooth.

200. Derivation of the Involute.—Let O and O', Fig. 186, be the centers of two disks, A and B, which are connected by a flexible band FG. If the disks are made to rotate on their axes in the direction shown by the arrows, the band will be unwound from disk B and wound up on A.

Imagine a marking point fixed to the band and moving in the line FG as the disks revolve. The point will move outward from F, tracing the curve F_3R on an extension of disk B, at the same time it will move inward on A from some point L, tracing the curve LG.

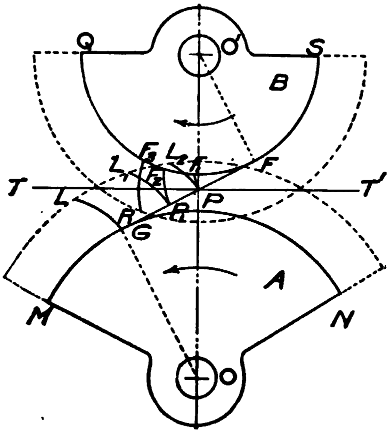


Fig. 186.

QS and MN correspond to the base circles in a pair of involute spur gears. P, the intersection of the band with the line of centers, is the pitch point. The pitch circles will pass through this point.

LG and F_3R are involutes of the base circles QS and MN and are the curves which form the outlines of involute gear teeth.

The base circles are always tangent to FG, hence their size is determined by the angle that this line makes with the tangent to the pitch circles. This angle is called the obliquity.

With a given pair of pitch circles an indefinite number of pairs of base circles may be drawn by changing the slope of the line FG.

The velocity ratio of the base circles is the same as that of the pitch circles within which they are drawn.

201. To Construct an Involute Curve.—Let *O*, Fig. 187, be the center of a base circle an involute of which is to be found. Lay off on the circumference of the base circle the arcs $P-1$, $1-2$, $2-3$, etc., equal or unequal and of any convenient length. Draw radii and tangents to the circle at these points. On the tangent perpendicular to $O-1$, rectify the arc $P-1$, locating the point *A*. On the next tangent rectify the arc $P-2$ locating the point *B*. Proceed the same with each tangent locating as many points as may be desired. A smooth curve drawn through the points *P*, *A*, *B*, *C*, will be the required involute. The accuracy of the work will be increased by taking the points *P*, 1, 2, 3, close together. Only that portion of the involute will be used which lies between the base and addendum circles of the gear.

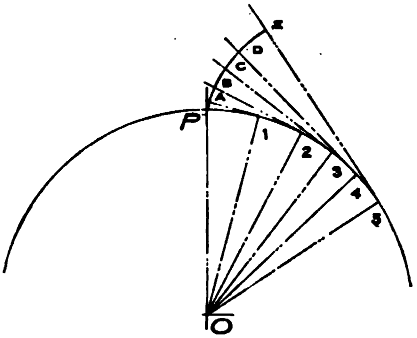


FIG. 187.

202. Obliquity and Angle of Contact.—In Fig. 188 let *O* and *O′* be the centers of two-pitch circles *LS* and *MN* which are in contact at *P*. Through *P* draw the tangent *TT′* and then draw *FPG* making a given angle with *TT′*. Tangent to *FG* draw the base circles *AB* and *CD*. Suppose *O* to be the driving axis and the direction of motion as shown by the arrows.

FG is in this case the locus of the point of contact between any pair of teeth, or if the direction of rotation be reversed the line will take the position *G′PF′*. The pressure between the teeth acts constantly in the direction of this line which is called the "line of pressure," or the "line of action."

FPT and *GPT′* are the angles of obliquity. These angles are equal and constant for involute gears their values being either 15 or 20 degrees as commercially made. The 15 degree angle is sometimes modified to 14½ degrees for convenience in making calculations because the sine of the latter angle happens to be the convenient decimal 0.25.

If the addendum of the teeth is known the addendum circles may be drawn from the centers O and O'. These addendum circles will cut the line of action in points as J and K and JPK will be the "path of contact." PJ is the arc along which the point of contact moves in approaching the line of centers and PK is the arc along which it moves in receding from the line of centers. These two arcs are called the arcs of approach and recess and the angles POJ and $PO'K$ are the corresponding angles of approach and recess.

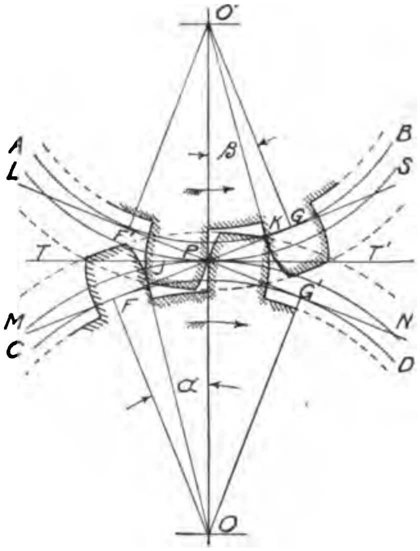


Fig. 188.

The angles of approach and recess should be greater than the angles of obliquity and the addendum lines should either pass through the points F and G or cut the line of action between F or G and the point P.

If the teeth shown be supposed consecutive, $POJ+PO'K$ will be approximately equal to the pitch angle. If the pitch angle is made greater than the sum of the angles of approach and recess, the pair of teeth on the right of the line of centers will cease contact before the pair on the left begin contact. The result will be an intermittent motion of the follower with a variation in the velocity ratio. In practice the angle of contact is made from one to one and one-half times as large as the pitch angle. The arcs subtended by these angles on the pitch lines will, of course, have the same relation.

203. Interference.—In Fig. 189 *A* and *B* are teeth of an involute pinion; *C* is the tooth of a rack and *D* that of a gear that mates with the pinion. *P* is the pitch point and *JJ′* and *KK′* are the lines of pressure. *F* and *G* are the points of tangency between the base circles and the pressure lines.

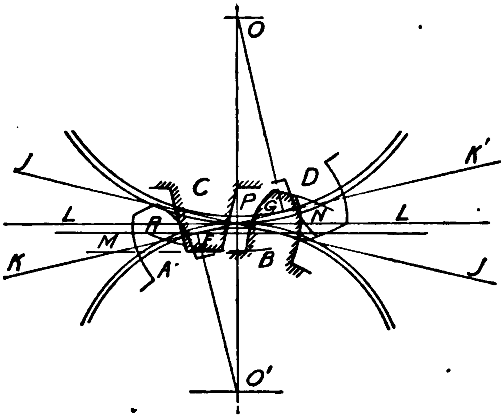


Fig. 189.

F and *G* are called the limit points. All contact between the teeth is on the line *FPG*, hence there can be no useful contact inside the base lines. A space must be cut out, however, below the base lines to allow the teeth to pass each other. It is customary to make the sides of the gear and pinion teeth inside the base circle radial straight lines.

When the addendum line of the rack tooth *C* or the gear tooth *D* intersects the line of pressure between *P* and *F* there will be no interference between the teeth. In the figure the rack addendum cuts the line of pressure at *M* and from *M* to *F* there will be interference between the face of the rack tooth and the flank of the pinion tooth, the latter being cut away as shown. To avoid this interference the rack tooth should be cut off on the line through the interference point.

The addendum line of the gear tooth cuts the line of pressure at *R* and from *R* to *F* there will be interference between the flank of the pinion tooth and the face of the gear tooth. To the right of *P* the addendum line of the pinion tooth *B* cuts the line of pressure at *N* and from *G* to *N* there will be interference between the face of the pinion tooth and the flank of the gear tooth, the latter being cut away as shown. This interference is corrected by changing the face curves so as to make the points of the teeth narrower.

204. Least Involute Pinion.—By Art. 202 the arc of action should never be less than the pitch arc. The angle of obliquity is generally 15 and never more than 22½ degrees. With these conditions fixed, there is a certain minimum number of teeth that a pinion may have below which it will fail to work with an involute gear.

Fig. 190 shows the limiting case in which the line of allowable contact, $FP+PG$ equals the pitch arc FD. The angle of obliquity GPT is supposed to have the maximum allowable value. Now if the number of teeth be decreased the points *F* and *G* will fall so close to *P* that the line *FG* will be shorter than the pitch arc. Hence the gears will fail to work.

The construction shown is that of a pair of 12-tooth pinions with standard teeth. It will be noticed that the points *M* and *N*, which are the theoretical limits of contact, are outside the interference points *F* and *G*, hence these teeth will need to be corrected for interference.

It follows from this discussion that a pair of 12-tooth pinions are about the smallest that can be made to work together. This pinion is therefore called the base of a set of standard interchangeable involute gears. Every gear in the set must have its teeth corrected so that contact will not take place beyond the interference point of this pinion.

If stub teeth be used, the addenda being shorter, the intersections *M* and *N* will be nearer to *F* and *G* and less correction for interference will be required.

If the obliquity be increased beyond the allowable limit for standard teeth, the number of teeth in a pair of mating pinions can be reduced to five. The teeth of these pinions will come to a point on the line of contact inside the interference point. Five-tooth pinions are then the smallest that can be made to work together with any obliquity. A four-tooth pinion will work with

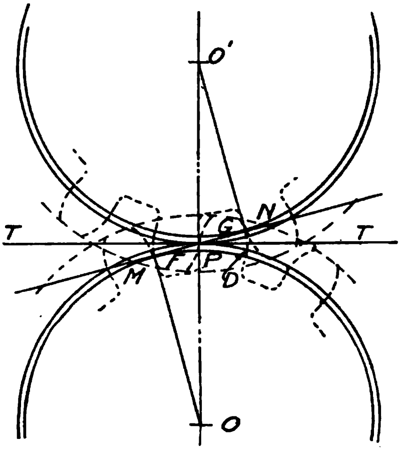


Fig. 190.

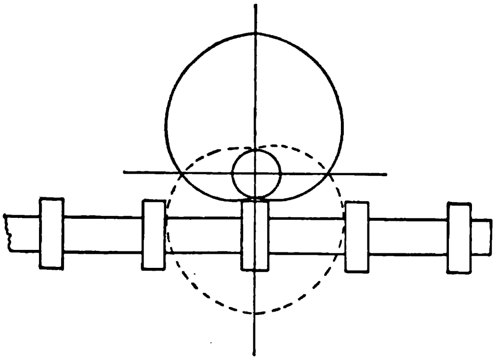


Fig. 191.

five or any greater number; a three-tooth pinion will work with any number greater than fourteen or with an annular gear of any number above three. Pinions of less than three teeth will not work at all on lines of action in the same plane.

If, however, the teeth be placed in parallel planes instead of in the same plane a two-leaved pinion can be made to drive a wheel or rack, the arrangement being shown in Fig. 191. These pinions are generally made with cycloidal teeth the mating gear teeth being radial straight lines as shown; this is, however, not necessarily the case for the pinions can be made just as well with involute teeth. The angle of obliquity is large and as the driver has only faces and the follower only flanks contact takes place only on the arc of recess.

205. Laws of Contact For Involute Teeth.—Involute teeth may be used either on external or internal gears, the axes rotating either in the same or opposite directions.

For a given pair of base circles the line *FG*, Fig. 188, always intersects the line of centers in a fixed point as *P*. Hence, the velocity ratio is constant. Now if the distance between the centers of these base circles be made either greater or less, *FG* will still intersect the center line in a fixed point and the velocity ratio will remain unchanged. This is one of the chief advantages of involute gears, viz., that within certain limits the velocity ratio is not affected when the gears are not properly adjusted.

Suppose the point of contact to move from *J* to *K*, along the line *FG*. Then it will be located on the line of centers only at the instant of passing *P*. From the condition of pure rolling contact it follows that the action is pure rolling at this instant only. To the right or left of this point the contact is mixed rolling and sliding.

The line *FG* is always normal to the tooth curves and as this line seldom makes a greater angle than 20 degrees, with the tangent to the pitch circles, it cannot pass through either of the centers O or O', the teeth will therefore drive each other positively.

206. Involute Gear and Pinion.—Suppose it is required to lay out a three-pitch involute gear, having 30 teeth, to mesh with a pinion of 24 teeth.

The pitch diameter of the gear will be $\frac{30}{3} = 10$ in. and the pitch diameter of the pinion $\frac{24}{3} = 8$ in. Draw the center line OO', Fig. 192, and locate the centers of gear and pinion 9 in. apart. Draw the pitch circles LS and MN and the tangent TT'. Lay off GPF making an angle of 14½ degrees with TT' and then draw in the base circles AB and CD. The tooth outlines will be involutes of these circles, and the velocity ratio of the gears will be to each other inversely as their radii. The diameter pitch being 3, the circular pitch will be $\frac{3.1416}{3} = 1.047$, and, assuming that the teeth and spaces are to have the same width, the thickness of the teeth will equal .523. These can now be laid off on both pitch circles. (Notice that the teeth are spaced off on the *pitch* and not on the *base* circles.)

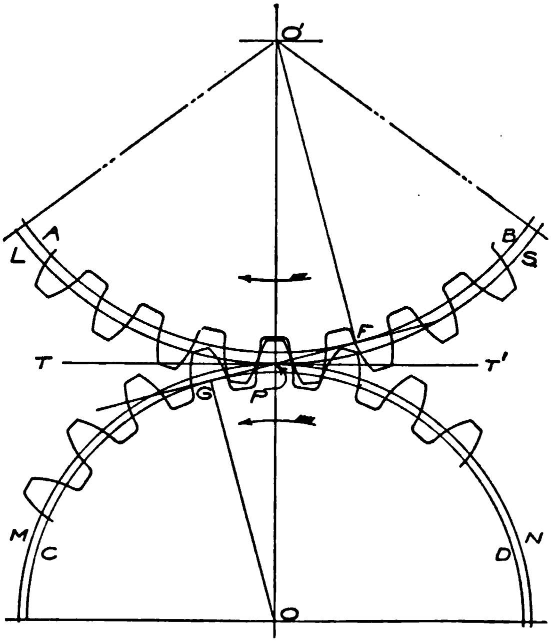


FIG. 192.

If the teeth are of standard length, the addendum will be $\frac{1}{3} =$ 0.33 in. and the dedendum will be 0.33 plus $\frac{0.33}{8} = 0.37$, about, for both gears. Draw in the addendum and dedendum circles. On *AB* and *CD* construct involutes by the method of Art. 201. Take a piece of thin wood or stiff cardboard a little longer than the radius *MN* and form upon it a templet of the part of the involute between the base and addendum circles. Fasten the templet at *O* and rotate it, drawing the curve through every other division point on the pitch circle, thus forming one side of the teeth. Turn the templet over, rotate it in the opposite direction and draw in the other side of the teeth. Construct a new templet and repeat this process for the pinion *LS*.

As no involute action is supposed to take place inside of the base circles, the spaces inside of these lines are simply notches cut to prevent interference with the faces of the opposing teeth. The sides of these spaces are radial straight lines. The fillets are standard size, drawn tangent to the dedendum circle and the radial flanks of the teeth.

207. Involute Rack and Pinion.—Let *O*, Fig. 193, be the center of a pinion which is to drive the rack *A* in the direction indicated. Suppose the pinion has 17 teeth, of 2½ pitch, then its diam-

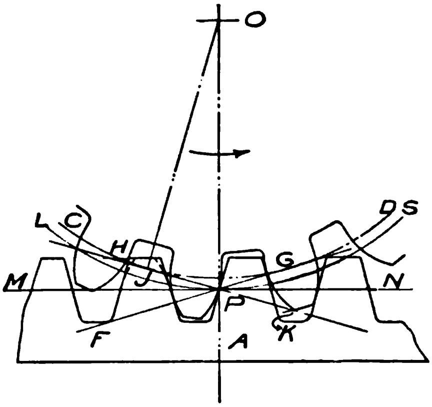


FIG. 193.

eter will equal 17÷2.5=6.8 in. Draw the pitch circle *LS* and take the line of pressure at an angle of 14½ degrees with the tangent. Draw the base circle *CD*. Lay off the circular pitch equal 3.1416÷2.5=1.25 in., and construct the pinion teeth as in Art. 206.

The pitch line of a rack is a circle of infinite radius; the base and pitch lines coincide and the involutes become straight lines, perpendicular to the line of pressure. Draw the pitch line *MN*, tangent to the pitch circle of the pinion, lay off upon it the tooth spaces $=1.25 \div 2$ equals 0.625, and through the division points draw the sides of the rack teeth. With the pinion driving as shown, contact will begin at *H* and end at *K*.

From *H* to *J* the point of the rack tooth will interfere with the flank of the pinion tooth. To avoid this the rack tooth must be rounded off somewhat as shown by the dotted outline.

The number of teeth in the rack will depend upon the extent of motion required. The addendum, dedendum, clearance and fillets on the rack are calculated the same as for the pinion.

208. Cycloidal Gear Teeth.—If a circular wheel be rolled along a smooth level track, a point on the rim will describe a cycloidal curve. The curve may be constructed graphically by rolling a circle on another circle or on a straight line.

The curve has three forms:

(a) The epicycloid formed by rolling one circle on the outside of a second circle.

(b) The hypocycloid, formed by rolling one circle on the inside of a second circle.

(c) The cycloid, formed by rolling a circle on a straight line.

The advantages of making all gear teeth, which are of the same size and pitch, so as to be interchangeable, together with the question of economy, make it undesirable to have two forms of teeth. So while the cycloidal tooth is the earlier form and is still in use to some extent, it has been largely displaced by the involute. It is doubtful whether the cycloidal tooth has any distinct advantages over other forms; its extensive use was probably due to the prominence given it by the early writers as a convenient example of the rolled-tooth outline.

The disadvantages are that the tooth is difficult and expensive to make and that two cycloidal gears will not work properly if the centers are not accurately adjusted.

The form is chiefly interesting here as an illustration of gear-tooth action.

209. Construction of the Epicycloid.—An approximate method of constructing an epicycloid is shown by Fig. 194. With the dividers, step off on AA' a series of small chords $P1$, 1 2, 2 3. Then step off on BB' the same length of chord $P1'$, $1'$ 2, $2'$ $3'$. As circle BB' rolls around on AA' the point $1'$ will fall at 1, $2'$ will

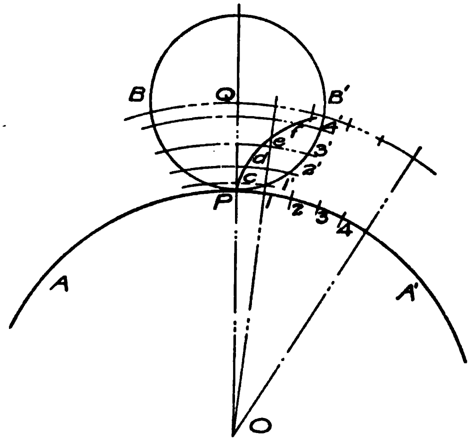


FIG. 194.

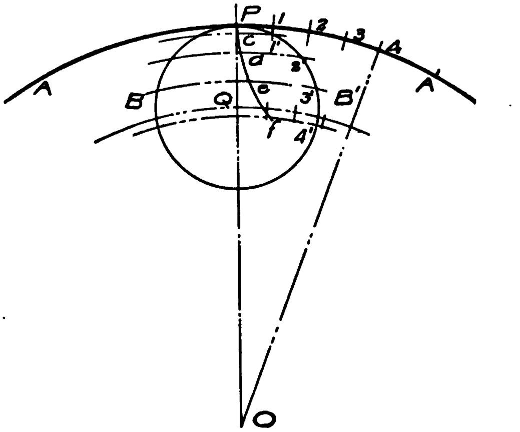


FIG. 195.

fall at 2 and so on. On O as a center, describe arcs through the points $1'$, $2'$, $3'$. Then with $1'P$ as a radius and point 1 as a center, make an intersection at c on the arc drawn through $1'$, again with $2'P$ as a radius and 2 as a center make an intersection at *d* on the arc drawn through 2′ and so on. A curve through points *Pcdef* will be an epicycloid, for evidently the point of contact at *P* between *AA′* and *BB′* will move away from the center *O* just as far as the points 1′, 2′, 3′ approach it. The approximation here consists in assuming that the small arcs *P*1′, 1′2′, 2′3′, are equal to the arcs *P*1, 1 2, 2 3, an error which is not appreciable if the arcs are made small.

210. Construction of a Hypocloid.—In Fig. 195, if circle *BB′* is rolled around on the inside of *AA′* the curve *Pcdef* described by a point *P* on *BB′* will be a hypocycloid. The method of construction is exactly like the previous case, except that the arcs drawn through 1′, 2′, 3′, 4′ fall inside of *AA′* instead of outside.

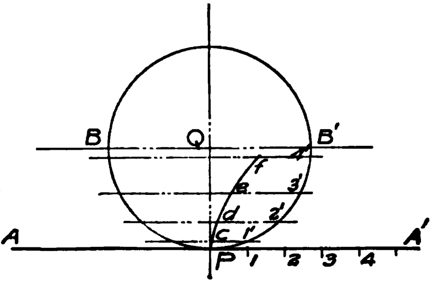


FIG. 196.

211. Construction of a Cycloid.—In Fig. 196 let the circle *BB′* roll on the straight line *AA′*, then the curve *Pcdef*, described by a point *P* on *BB′*, will be a cycloid. In this case, draw parallel straight lines through the points 1′, 2′, 3′, 4′. With 1′*P*, 2′*P*, 3′*P*, 4′*P* as radii, and with centers 1, 2, 3, 4, on *AA′* find the intersections *c*, *d*, *e*, *f*, and draw the curve.

212. Forms of the Cycloidal Tooth. (a) *Epicycloid and Hypocycloid.*—In Fig. 197 if a circle with center at *Q* be rolled around on the outside of *MN*, a point *P* on its circumference will trace an epicycloid *FP′*. This is the form of curve used for the outlines of the addenda of cycloidal teeth. If *Q* be rolled on the inside of *LS*, the path of *P* will be a somewhat flatter curve *GP′*; this curve forms the outlines of the dedenda of mating teeth.

(b) *Epicycloid and Radial Line.*—If the relative diameter of the describing circle is increased, as in Fig. 198, the curve *GP′*

will become more flat. When the diameter of the circle is made just equal to the radius of *LS*, the point *P* will move back along the line *PO* and *GP′* will become a straight line. From this it follows that whenever the diameter of the describing circle is made equal to the radius of the pitch circle, the flanks of the teeth are radial straight lines. If an epicycloid and radial line generated in this manner be used for the faces and flanks of teeth, the action between them will be precisely like that of the epicycloid and hypocycloid of Fig. 197.

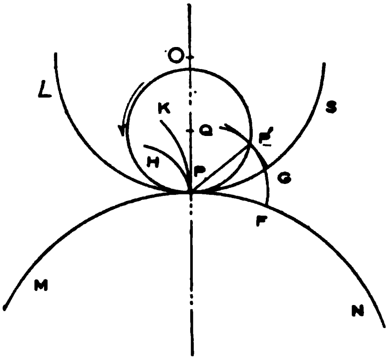


Fig. 197.

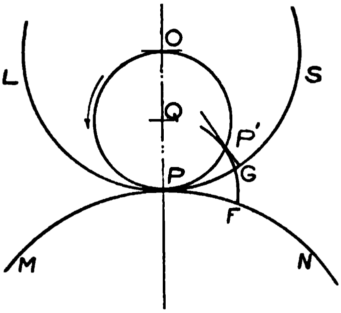


Fig. 198.

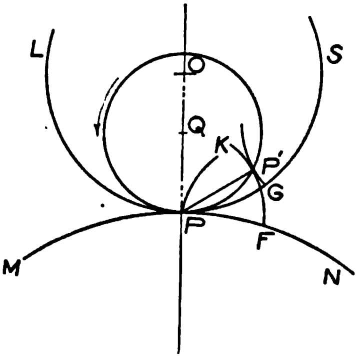


Fig. 199.

(c) *Epicycloid and Pin.*—If the diameter of the describing circle is made greater than the radius of *LS* as in Fig. 199 the curve PK or its equal GP' will lie on the right hand side of the center line and become convex in the opposite direction. Teeth with curves like PK for flank outlines are said to be "undercut," that is, they are narrower at the base than at the pitch line. In action they still conform to the laws of contact for rolled tooth curves. As the diameter of the describing circle is further increased and approaches that of the pitch circle, the hypocycloid reduces to a point at P. If a pin be placed at P, the corresponding epicycloid described by Q on MN will act with it fulfilling the laws of contact as if the pin were a hypocycloid or a radial line.

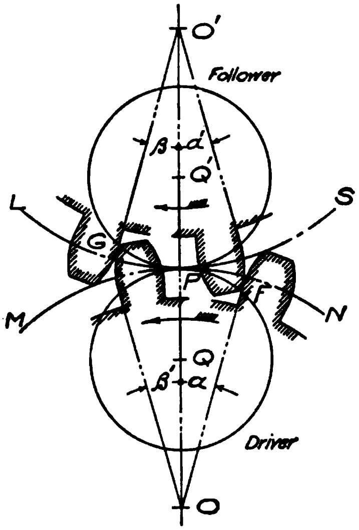


FIG. 200.

213. Relation Between Pitch, Obliquity and Describing Circles.—To design a cycloidal tooth, three factors must be known: (1) The pitch; (2) the obliquity of the action; (3) the size of the describing circles.

These factors have been determined by experience and their practical limits fixed. The pitch depends upon the strength required for the teeth. The maximum obliquity is made the same for all standard gears and the size of the describing circle depends upon the obliquity and is limited by the size of the gear.

214. Length of the Pitch Arc.—In Fig. 200 $FP+PG$ is the path of contact. Let this path be equal in length to the circular pitch. If the teeth shown are consecutive ones, contact will begin at F at the same instant that it ceases at G and the action will be continuous. If FP and PG are made shorter, the pitch remaining the same, one pair of teeth will cease contact at G before the next pair begins at F, in which case the contact will not be continuous and the velocity ratio will vary.

If the arcs FP and PG be lengthened, the circular pitch remaining the same, the pair of teeth on the right will begin contact some time before the pair on the left ceases. Contact in this case will be continuous and the velocity ratio will not vary. It follows, therefore, that the arc of contact may be greater, but should never be less than the circular pitch. In standard cycloidal gears it is made from one and one-half to two times this pitch.

215. Angle of Obliquity.—In Fig. 201, *PG* and *PF* are the lines of pressure at the instant when one pair of teeth is beginning comtact on the right and another pair quitting contact on the left. The angles *GPT′* and *FPT* are the maximum angles of obliquity. In cycloidal gears, these angles, which may or may not be equal, are constantly varying between a maximum value when the contact is at *F* or *G*, and zero when the contact is at *P*. The smaller this angle of obliquity, the smoother will be the action between the teeth and the smaller will be the component of the tooth pressure in the direction of the center line of the gears. Both of these results are desirable hence the angle of obliquity should be kept as small as posible. It is also true that the action between the teeth will be smoother near the point *P* than at the ends of the path of contact.

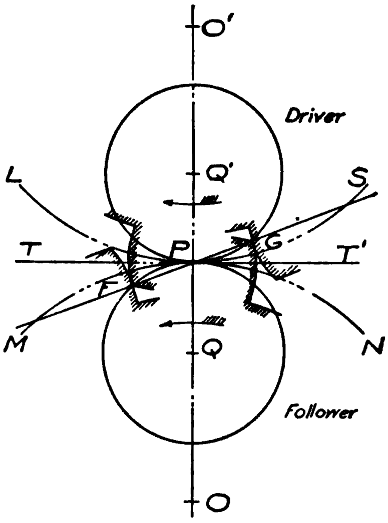


Fig. 201.

In commercial gears the maximum angle of obliquity is about 15 degrees for the cycloidal system; 30 to 36 degrees is the upper limit of this angle if the gears are to work properly.

216. Describing Circles.—With a given pitch, the size of the describing circles and the maximum angle of obliquity are mutually dependent.

In Fig. 202, let *O* and *O′* respectively be the centers of a driver and follower where *Q* and *Q′* are the centers of the corresponding describing circles and *FPT* and *GPT′* are the maximum angles of obliquity. *F* and *G* are the points where the teeth begin and end their contact, *PF* and *PG* are the lines of pressure at these points and *AA* and *A′A′* are the addendum lines as determined from the pitch.

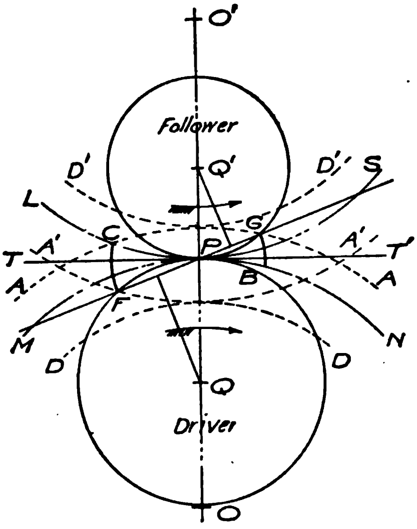


FIG. 202.

It will be seen that the point *F* is located at the intersection of describing circle *Q* with *A′A′*, the addendum line of circle *O′*, and the line of pressure. Similarly *G* is located at the inter-section of describing circle *Q′* with *AA*, the addendum line of *O*, and the line of pressure.

If any two of the three factors, describing circles, addendum lines or pressure lines are known, the points *F* and *G* can be located and the other factor determined.

(a) When the angles of obliquity and the pitch are known, calculate the addendum from the known pitch and draw the addendum circle. Lay off the angles of obliquity. Then the points *F* and *G* will lie at the intersections of the pressure lines and the addendum lines.

(b) When the angles of obliquity and the describing circles are known the latter may be drawn about the centers *Q* and *Q′* and their intersection with the lines of pressure will give the points *F* and *G*. The addendum circles may then be drawn through these points. Since the pitch circles pass through *P*, the height of the addendum is determined and the pitch may be calculated from (d) of Art. 197.

(c) When the pitch and the describing circles are known, calculate the addendum, draw the addendum circles and locate *F* and *G* at the intersection of the addendum circles and the describing circles. The lines of pressure may then be drawn through *PF* and *PG*.

217. Limiting Size of Describing Circles.—In Fig. 203, *FPE* and *DPC* are the outline curves of a pair of mating teeth. *PE* is generated by rolling describing circle Q′ to the left on the outside of *MN*. *PD* is generated by rolling the same describing circle to the left on the inside of *LS*. *PF*, the dedendum of the curve at *PE*, is generated by rolling *Q* to the right on the inside of *MN*. *PC*, the addendum of the curve at *PD*, is generated by rolling *Q* to the right on the outside of *LS*. Therefore faces and flanks which work together are generated by the same describing circle rolling in the same direction with respect to the pitch point.

The figure shows the form of the flank when the describing circle equals, is less than, or greater than, the radius of the describing circle within which it rolls. The form *PD″* is generally undesirable in commercial gear teeth and by Art. 212 this flank is generated by a describing circle greater in diameter than the radius of the pitch circle *LS*, within which it rolls. Hence it follows that the diameter of the describing circle should never be greater than the radius of the pitch circle. Fig. 202 shows that as the diameter of the describing circle decreases the angle of obliquity increases, but by Art. 215 this angle should be about 15 degrees, hence the limit below which the size of the describing circle cannot be decreased is fixed.

The action of a pair of gear teeth being less smooth on the arc of approach than on the arc of recess, the arc of approach may be eliminated altogether by making the describing circle of the driver's flank equal to zero. The teeth of the driver will in that case have only faces and those of the follower only flanks. Contact will take place only on the arc of recess from P to G, Fig. 204, which shows the form of the teeth.

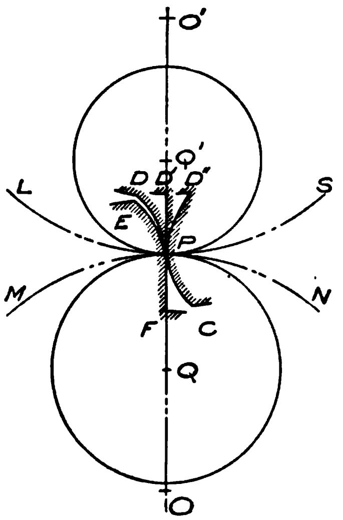


Fig. 203.

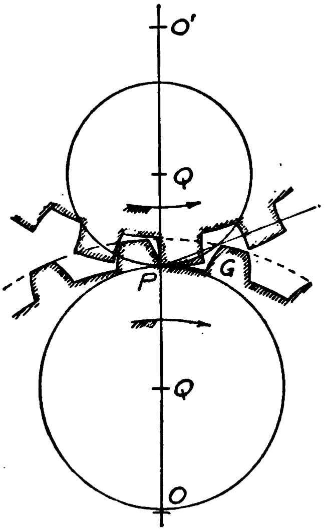


Fig. 204.

218. Laws of Contact for Cycloidal Teeth.—In Fig. 205, let A be a directrix of any shape capable of being rolled on another line. Let LS and MN be two pitch lines having centers of rotation at O and O'. Let the pitch lines and directrix have a common point of contact at P. Roll LS, MN and A, simultaneously upon each other. The point P considered as a point on MN will move to F. As a point on LS it will move to G and as a point on A it will move to P'. A marking point placed at P would trace a curve FP' on an extension of MN and at the same time it would trace the curve GP' on LS. The curves FP' and GP are conjugates.

If used for the outlines of gear teeth and *FP'* be made to drive *GP'* the point of contact between them will remain constantly on *A* and trace the curved path *PP'*.

Since *P* lies between *O* and *O'*, the rotation of the curves, and also their pitch lines *LS* and *MN*, about these centers, will be opposite in direction.

P may be taken as the instant center of rotation of any point on *A*. Connect *P* and *P'* by a straight line. The direction of motion of *P'* at any instant will be that of *P'T* perpendicular to *PP'*. Then *P'T* is the common tangent to the curves at that instant and *PP'* is the common normal. Since *P'* is any point of contact all normals to the curves from points on *A* will pass through *P*. If *P* is fixed in position, that is, if *OP* and *O'P* are of constant length, the velocity ratio of *LS* and *MN* will be constant.

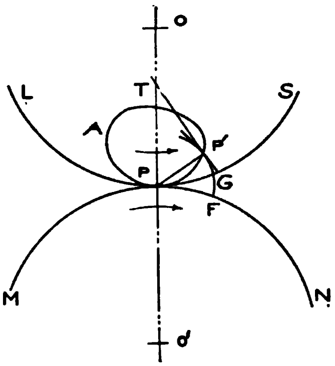


Fig. 205.

As the curves *FP'* and *GP'* revolve about the centers *O* and *O'*, the point of contact is located for a single instant on the line of centers and at that instant the contact is pure rolling. When the point of contact is to the right or left of the line of centers as at *P'*, the motion of this point may be resolved into two components one tangent and one normal to the curves, hence by Art. 43 such contact is mixed rolling and sliding.

It is apparent from the figure that a common normal to the curves from any point on *A* can never pass through the centers *O* and *O'*, hence the driving action is positive.

The indefinite general curve *A* is replaced by a circle in generating curves corresponding to *FP'* and *GP'* for the outlines of cycloidal teeth.

219. Least Follower.—Sometimes it is necessary to find the smallest follower that will work with a given pinion. Let *O*, Fig. 206, be the center of a known pinion, having radial tooth flanks, and let the circular pitch *PF* be equal to the arc of recess.

Lay off *FK* equal to one-fourth *PF*, draw *OK* and produce it indefinitely. *OK* will pass through the center of a tooth on *O*, and when contact ceases the point of this tooth will be just leaving the flank of the followers' tooth, at the end of the arc of recess. Call this point *G*. An epicycloid of *MN* with origin at *F* may be made to intersect *OK* in the same point. To locate *G*, draw a tangent *TT′* at *P* and rectify the arc *PF* at *PH*. Take *PV* equal to one-fourth *PH*, and with *V* as a center and *VH* as a radius, draw an arc cutting *OK* at *G*. This arc will closely approximate the true epicycloid.

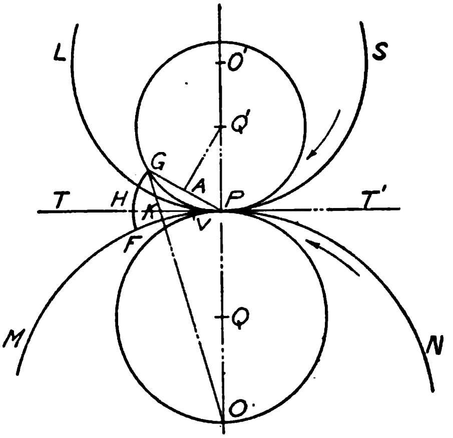


Fig. 206.

To find the describing circle and pitch circle, draw *PG*, bisect it at *A*, and erect a perpendicular cutting the line of centers at *Q′*. On *Q′* as a center, draw a circle through *P* and *G*.

This will be the required describing circle and the arc *PG* which equals *PH* or *PF* is the arc of recess. Make the diameter of the pitch circle twice that of the describing circle for a trial. This will give the follower radial flanks. Multiply the trial diameter by the pitch, and if the result comes out a fraction for the number of teeth, make it equal to the nearest integer. Divide the integer by the pitch to find the true diameter. The true diameter will vary so little from the trial diameter that the size of the describing circle need not be changed.

220. Least Follower for Standard Teeth.—If the teeth are to be standard size, the least follower for a given gear will be larger than when the gear teeth are pointed. Assume the driver to have radial flanks as before. Then with the diameter of the driver and the pitch given, we can calculate the addendum and draw the addendum circle of the driver. At the end of the arc of recess, which must also be known, the advancing corner of the driver's tooth will be located at the intersection of the addendum circle of the driver with the describing circle in the follower, G Fig. 207. Draw PG and erect a perpendicular at its middle point, cutting the line of centers at Q', the center of the required describing circle. Find the pitch circle of the follower as in the previous article.

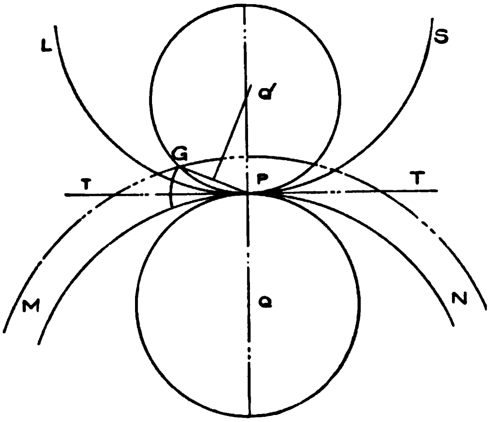


Fig. 207.

221. To Lay Out a Cycloidal Gear and Pinion.—Two shafts 30 in. apart, one turning at the rate of 200 revolutions per minute, and the other 100, are to be connected by a pair of cycloidal spur gears. Suppose the power to be transmitted requires the gears to be two-pitch. By Art. 40, Rule III, the velocity varies inversely as the radii. The sum of the radii is thirty inches. Let the pinion be the driver, then $\frac{\text{Rad. Driver}}{\text{Rad. Follower}}=\frac{100}{200}$. In Fig. 208 ($a$) lay off OO' equal 30 in. and divide it into three equal parts and take OP equal to two of these parts, then OP

=20 in., is the radius of the follower and $O^1P=10$ in., is the radius of the driver. With OP and $O'P$ as radii draw the pitch circles MN and LS.

By Art. 197, $N=P\times D_p=2\times 10=20$ for the driver, and $2\times 20=40$ for the follower.

$$A=\frac{1}{2}\text{ inch, and } C=\frac{1}{8}\times\frac{1}{2}=\frac{1}{16}$$

$$D=A+C=\frac{9}{16}\text{ in.}$$

The circular pitch equals $\frac{\pi}{P}=\frac{3.1416}{2}=1.57.$

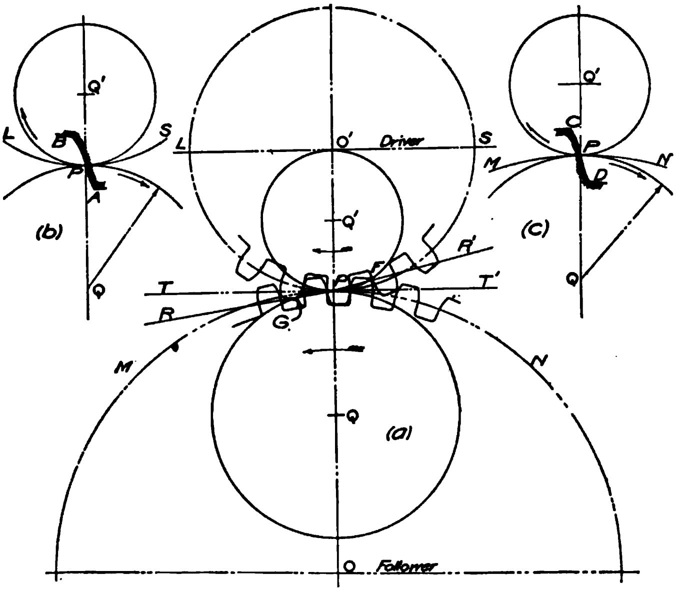


FIG. 208.

Draw the addendum and root circles and divide the pitch circle of the driver into 40, and the pitch circle of the follower into 80 spaces, $\frac{1.57}{2}$ in. long. Draw TT' and lay off RR' making with TT' an angle of 15 degrees. RR' intersects the addendum lines in F and G, the points of beginning and of quitting contact.

Bisect chord *PF* and erect a perpendicular cutting the line of centers at *Q'*, and with *Q'* as a center and radius *Q'P*, draw one describing circle. In a similar manner construct the other describing circle on the chord *PG*. Roll *Q*, Fig. 208 (*b*), to the right on the outside of pitch circle *LS* forming the epicycloid *PA*. Roll *Q'* in the opposite direction on the inside of circle *LS* forming the hypocycloid *PB*. Make a templet to fit the curve *APB*, fasten the templet at the center of *LS* and rotate it counter-clockwise. Draw *APB* through every second tooth space as laid out on the pitch circle. Then turn the templet over and go around in the opposite direction, completing the tooth outline.

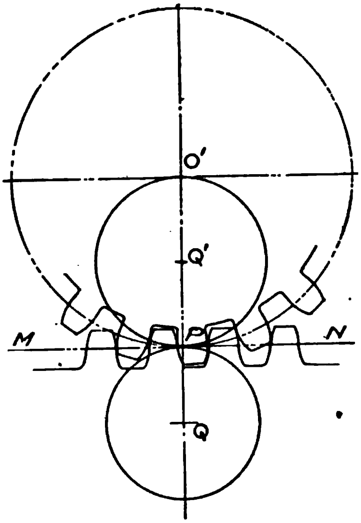


FIG. 209.

Roll circle *Q'*, Fig. 208 (*c*), to the left on the outside of pitch circle *MN*, tracing the epicycloid *PC*. Roll *Q* in the opposite direction on the inside of *MN* forming the hypocycloid *PD*. Make a new templet for *CPD* and draw in the tooth outlines as before. Finally draw in the fillet tangent to the root circle and flanks of the teeth.

In practice the curves must be laid out the exact size of the tooth curves required.

222. Cycloidal Rack and Pinion.—Fig. 209. The pitch line *MN*, of a rack, is straight. The tooth faces are cycloids. They are determined by rolling upon the pitch line, the same describing circle that would be used for the flanks of a mating pinion. The tooth flanks are also cycloids, but they are found by rolling upon the pitch line the describing circle that would be used to form the faces of a mating pinion.

To lay out a rack, first space off the teeth along the pitch line. Next roll out the tooth curves and make a templet. Move the templet along the pitch line, keeping it parallel to its original position. Draw in the outlines for one side of the teeth, then reverse the templet and draw the other side.

223. An odontograph is a method or an instrument for simplifying the construction of tooth curves. Two kinds of odontograph methods are employed. (1) Approximating the true curves by circular arcs; (2) approximating the true curves by others which are more easily constructed than the true curves. Approximate methods of drawing teeth are of doubtful value, their advantage being wholly on the side of speed. When teeth are laid out for making patterns, templets or cutters, they should never be used. Cut teeth of fine pitch are shaped by accurately ground cutters so there is usually no need of laying them out at all. The odontograph is sometimes convenient when a large number of teeth must be shown on drawing.

224. Brown and Sharpe's Odontograph Methods.—(a) After the pitch, base, addendum and dedendum circles have been drawn and the tooth spacing laid off the tooth curves may be drawn by taking a radius equal to one-quarter of the pitch radius, Fig. 210, and with centers on the base circle drawing arcs to form the sides of the teeth. This gives a tooth that does not conform closely to the true involute and the method should never be used where any great degree of accuracy is required.

(b) *Involute Gears of Less than Thirty Teeth.*—Construct the faces of the teeth outside of the base circle as described under (a) and then draw the rest of the tooth as a radial straight line. This is not exactly the method described by the authors but it is near enough when the teeth are simply to be represented.

(c) *Cyclodial Gears With Radial Flanks.*—In interchangeable sets of gears the smallest pinion in the set, commonly a pinion of

15 teeth, has radial flanks, that is, the diameter of its describing circle is equal to the radius of the pinion. Such a gear is called the base of the set and its teeth may be laid out as follows, Fig. 211:

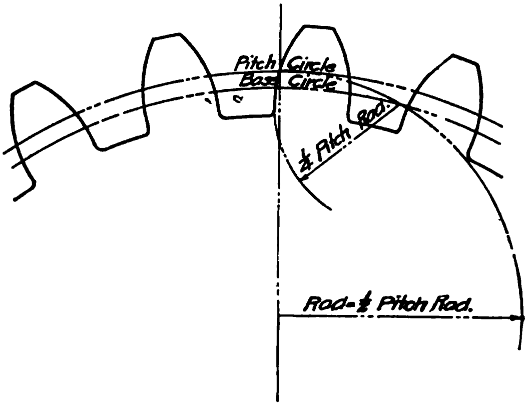


Fig. 210.

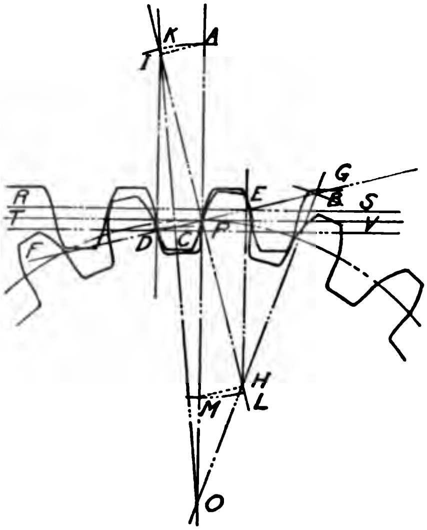
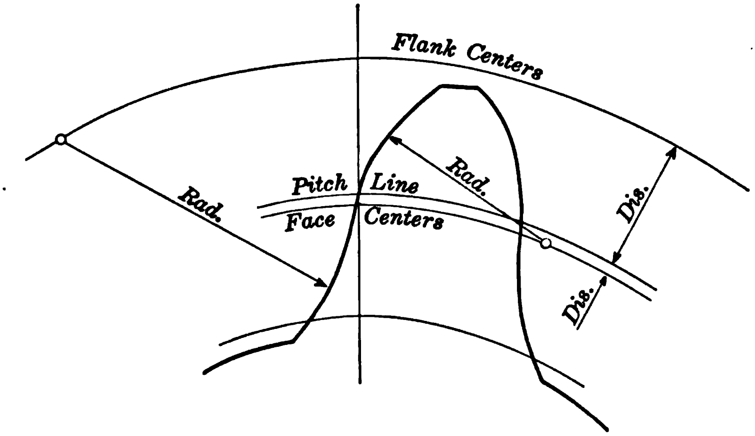


Fig. 211.

Draw pitch, addendum and dedendum circles and space off the teeth. From the center O through one of the tooth divisions draw the line *OPA*. With *P* as a center and with radius equal to the pitch radius of the gear draw arcs *AK* and *ML* and on these arcs lay off the thickness of the tooth. Erect the perpendicular, *MH* and *AI*. Draw the line *KPL* and perpendicular to it draw the line of pressure *FPG*. Draw the line *OI* and through



FOR CYCLOIDAL TEETH

No. of Teeth		*For One Diametral Pitch* *For any other pitch divide by that pitch.*			
Exact	*Intervals*	*Faces*		*Flanks*	
		Rad.	*Dis.*	*Rad.*	*Dis.*
10	*10*	*1.99*	*.02*	*-8.00*	*4.00*
11	*11*	*2.00*	*.04*	*-11.05*	*6.50*
12	*12*	*2.01*	*.06*	∞	∞

Fig. 212.

C, its intersection with the line of pressure, draw a circle concentric with the pitch circle. Locate the centers of the tooth curves on this circle. The radii of these curves will be equal to the distance from *C* to the further side of the next tooth on the right. The curve will end at the pitch circle, the rest of the tooth outline being a radial straight line.

(d) *Cycloidal Gears with Non-radial Flanks.*—In an interchangeable set of gears all except the base have curved flanks and the curved outline will change at the pitch circle. Draw pitch, addendum and dedendum circles and space off the teeth as before. Draw the arcs *AK* and *ML* from *P* as a center with radii equal to the radii of the smallest gear in the set. Draw *KPL* and the line of pressure *FPG* perpendicular to it. Erect the perpendiculars *MH* and *AI*. Now draw a line from *O* through *H* to intersect the line of pressure at *B*. On *O* as a center draw a circle through *B*. With centers on this circle, and with radius equal to *BD*, the distance from *B* to the nearest side of the first tooth beyond the line of centers, draw the flanks of the teeth. The faces are constructed as in the previous case.

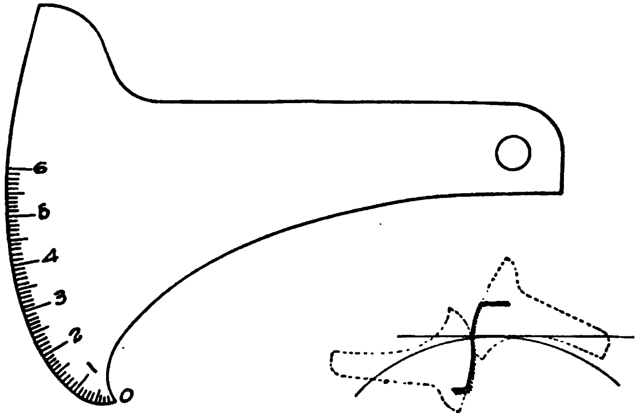


FIG. 213.

(e) *Cycloidal Rack.*—Lay out standard addendum, and pitch lines and space off the teeth. Draw the arcs *AK* and *ML* with radius equal to the radius of the mating gear or the smallest gear in the set. Lay off the width of the teeth on these arcs and erect the perpendiculars *AI* and *MH*. Draw *HK* and the line of pressure perpendicular to it. Drop perpendiculars to the pitch line at *H* and *I*. These perpendiculars will intersect the line of pressure at *D* and *E*. Through these points draw the lines *RS* and *TV* parallel to the pitch line and on these parallels locate the centers of the tooth curves. The radius of the face and flank will be the same and equal to the distance from *C* to the nearest side of the first tooth on the other side of the line of centers.

225. Grant's Odontograph Method is based on an older one devised by Prof. Willis and is one of the most convenient and accurate methods of approximating tooth outlines. In this method the tooth curves are drawn with circular arcs which have been worked out experimentally so as to pass through two or more points of the true outline. Tables of radii for these arcs and of loci for their centers are illustrated by Fig. 212. These tables are given for both involute and cycloidal gears and by their use tooth curves may be very easily and rapidly drawn.

226. Templet Odontographs.—A templet odontograph is an irregular curved piece some portion of which can be made to fit any required tooth outline. Fig. 213 is Robinson's odontograph, which is designed on the theory that every tooth curve can be made to conform to some part of a logarithmic spiral. A table of arbitrary factors accompanies the instrument and by their aid the tooth outlines are drawn directly on its edges.

PROBLEMS

1. A 4-pitch spur gear is 10 in. in diameter. Find the circular pitch, the module and the number of teeth.

2. If a gear has 36 teeth and a circular pitch of 1 in. find its diametral pitch.

3. Show wherein the 6 fundamental principles of Chapter III are true for involute tooth curves.

4. Prove that the epicycloid and the hypocycloid fulfil all the requirements for gear tooth outlines.

5. The circular pitch of a pair of involute spur gears 10 and 5 in. in diameter is 2+ in., the addendum being ¾ in. and the obliquity 14½ degrees. Locate the point of beginning and ending contact.

6. Find the size of the describing circles in problem 5 if the gears are cycloidal.

7. Find the angles of obliquity in problem 5 if the diameters of the describing circles equal the radii of the gears.

8. Two gears, 6 and 12 in. in diameter, have describing circles 3 and 5 in. in diameter. If the circular pitch is 2, and the arcs of approach and recess are each equal to ¾ of the circular pitch, find the addenda and the angles of obliquity.

9. A standard 3-pitch gear is 9 in. in diameter and has cycloidal teeth with radial flanks. It meshes with a gear 12 in. in diameter which also has radial tooth flanks. Find the number of teeth, velocity ratio, circular pitch, size

of the describing circles, angles of obliquity, addendum, dedendum, clearance, fillets and backlash.

10. Prove that a cycloidal gear has radial tooth flanks when the diameter of the describing circle equals the radius of the gear.

11. Prove that the sides of an involute rack tooth are straight lines.

12. What is the least number of teeth that can be given an involute pinion to work with a 3-pitch standard gear of 36 teeth.

13. Indicate the shape of the teeth on a pair of gears that have no arcs of approach.

14. Lay out a pair of 3-pitch involute gears for the following conditions: Diameter of gears 10 in. and 36 in. Standard teeth.

15. Lay out the same gear with stub teeth and an angle of obliquity of 20 degrees.

CHAPTER XI

DIRECT CONTACT. ROLLING AND SLIDING MOTION CONTINUED

ANNULAR, BEVEL, SKEW BEVEL, ELLIPTICAL, PIN, CHAIN, FACE AND IRREGULAR GEARS

227. Internal Gears.—If the teeth of a gear be made on the inside of the rim so that they extend inward toward the center of the wheel, the arrangement is an internal gear, Fig. 214. The mate to an internal gear is a spur gear, or pinion.

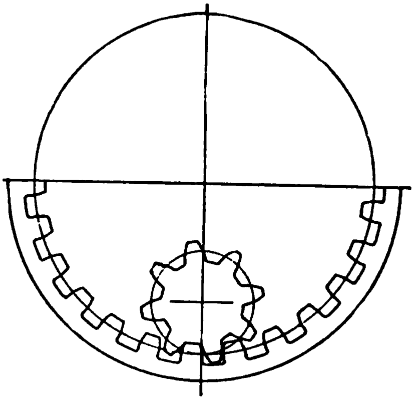


Fig. 214.

Internal gears may be constructed on either circular or irregular pitch lines. The circular pitch line is the most common and important form and an internal gear having a circular pitch line is called an "annular" gear.

The laws of contact and velocity ratio are the same for internal as for external gears.

An internal gear and its pinion will rotate in the same direction since the centers are both on the same side of the point of contact.

Either the pinion or the gear may be made the driver, but an internal gear cannot be made to drive a rack nor another internal gear.

The form of the teeth is precisely like that of the spaces on an external gear of the same size and pitch. Either the involute or the cycloidal tooth may be used, the methods of forming and making the teeth being in general the same as for external gears.

228. Design of an Annular Involute Gear.—Let *LS*, Fig. 215, be the pitch circle of a pinion and *MN* the pitch circle of a gear. Draw the line of pressure *GFP* and find the base circles as in the case of spur gears. Lay off the tooth spaces on the pitch circles and calculate a standard addendum for the pinion and a standard dedendum for the gear. Draw the corresponding addendum and dedendum circles.

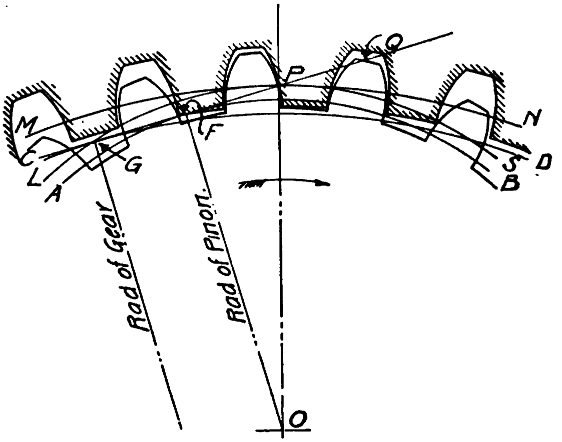


FIG. 215.

Since there can be no involute action inside of a base circle the points of the gear teeth are cut off so that they cannot make contact with the pinion teeth inside of the base circle of the pinion. Therefore, to find the line limiting the length of the gear teeth draw from the center of the gear a circle through F, the latter being the point of tangency between the pressure line and the base circle of the pinion.

The flanks of the pinion teeth are also theoretically limited by a circle through *F*. But as the spaces must be deep enough to accommodate the points of the gear teeth without interference, when the contact is in the vicinity of P, the flanks are prolonged with radial straight lines.

The correct way to find the depth of the spaces on the pinion is to roll one pitch line on the other and find the envelope of the points of the gear teeth. As this process involves considerable labor the proper amount of clearance may generally be determined by inspection.

The faces of the pinion teeth are constructed the same as for any other spur gear. The flanks of the gear teeth are made like the faces of an equivalent spur gear.

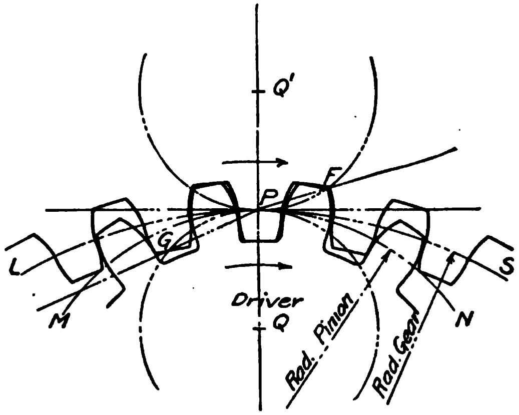


FIG. 216.

In the figure if the pinion is driving in the direction shown by the arrow, contact will begin at F and end at Q.

229. Annular Gears with Cycloidal Teeth.—The difference between the number of teeth on an annular gear and its pinion is generally large and involute teeth, made according to the previous article, will not cause trouble by interference. When, however, the gear and pinion are nearly the same size the cycloidal tooth may be the better. The teeth in this case should be laid out as for a spur gear except that the flanks of the teeth will be epicycloids and the faces hypocycloids. We shall then have two epicycloids and two hypocycloids mating with each other instead of epicycloids mating with hypocycloids, Fig. 216. Cycloidal gears may be so constructed as to give two paths of contact at the same time. This secondary contact, as it is called, may be an advantage in favor of the cycloidal form, as it allows the pressure between the teeth to be distributed over a larger area, also, since it takes place during the arc of recess, it gives a smooth action to the teeth.

230. Secondary Action of Annular Gear Teeth.—In Fig. 217, let O and O' be the centers of an annular gear and its pinion, let Q and Q' be the exterior and interior describing circles and let Q_3 be a secondary describing circle so chosen that its diameter is equal to the diameter of the pinion plus the diameter of Q or equal to the diameter of the gear minus the diameter of Q'.

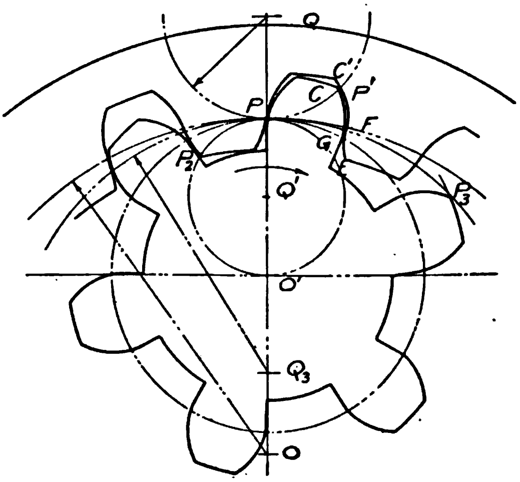


FIG. 217.

Now it can be shown that the epicycloid CG, which forms the face of the pinion tooth, can be generated by rolling either Q or Q_3 on the outside of the pitch circle of the pinion. Also the epicycloid $F\ C$ of the gear is generated by rolling Q_3 on the outside of the pitch circle of the gear.

P is the pitch point and P', P_2 and P_3 are intersections of the addendum lines of the gear and pinion and the describing circles Q, Q' and Q_3.

If the pinion is taken as the driver and motion is in the direction shown by the arrow, contact will begin at P_2, proceed along the arcs P_2, P, P' and cease at P'. From P_2 to P the contact is between the flank of the pinion tooth and the face of the gear tooth. From P to P' the contact is made between the face of the pinion tooth and the flank of the gear tooth.

At P there is established between the face of the pinion tooth and the flank of the gear tooth, a *secondary contact* and this contact proceeds along the arc $P P_3$ of the secondary describing circle Q_3. The reason for such secondary contact is that $G C$ is an epicycloid of the pinion, formed by either of the describing circles Q or Q_3. As an epicycloid formed by Q, it makes contact with FC' along the line PP', and as an epicycloid formed by Q_3, it makes contact with FC' along the line PP_3.

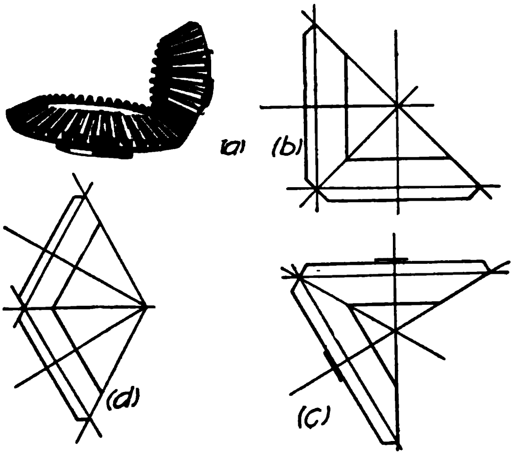


FIG. 218.

It can be shown that if the radius of Q is less than one-half of the center distance OO' the teeth will interfere. Again, if the radius of Q' is made greater than one-half of OO', the teeth will be separated so that no secondary action at all will take place. Hence, if annular gears are to have secondary contact, *the sum of the radii of exterior and interior describing circles must equal the distance between the centers of the gear and the pinion.*

231. Bevel Gears.—Fig. 218 (a), may be made to transmit motion between two axes in the same plane which intersect. The angle made by the axes may be, Fig. 218 (*b*), right; (*d*), oblique; or (*c*), obtuse.

When the angle between the axes is a right angle and the gears are the same diameter they are called "miter gears."

The pitch surfaces of bevel gears are generally made like the cones of Art. 53. They may, however, be elliptical, internal or irregular.

The tooth elements are commonly straight lines which meet in the vertex of the cone, the gears may also be made with helical or twisted teeth.

The laws of contact and velocity ratio are precisely like those of cylindrical gears having the same form of pitch line.

Bevel gears are extensively used for both light and heavy work and their advantages are obvious for connecting shafts which are not parallel.

232. Form of Bevel Gear Teeth.—In spur gears, if a tooth element be moved so as to generate a tooth surface, the element will always remain parallel to the axis of the gear. The end of the element will move in a plane which may be used as a plane of reference on which to study the tooth outline.

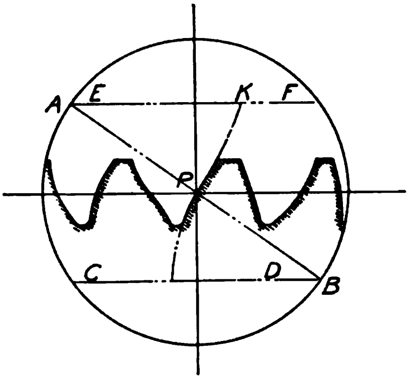


FIG. 219.

An element of a bevel gear tooth in generating the tooth surface rotates simultaneously in two directions about the vertex of the conical pitch surface. The end of the element will therefore move in the surface of a sphere. Hence the outlines of bevel gear teeth must be studied on a spherical surface of reference..

The outlines of commercial bevel gear teeth are of three forms: (1) involute; (2) octoidal; (3) cycloidal.

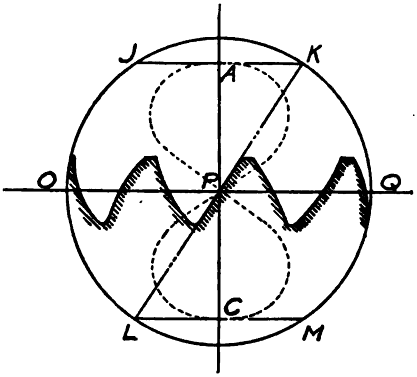


FIG. 220.

The spherical involute tooth is formed by rolling a great circle of a sphere tangent simultaneously to two small circles of the sphere, Fig. 219. *AB* is a great circle and *CD* and *EF* are the

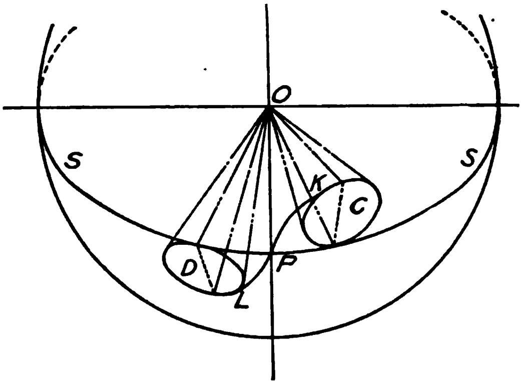


FIG. 221.

small circles. If *P* is the describing point it will trace the spherical involute *PK*.

If an involute rack be bent around in a circle, the result will be a crown gear, Fig. 220. The straight sides of the involute rack teeth will become arcs of great circles of the sphere *OQ*. These arcs form the basis for the mechanical generation of the outlines of octoidal teeth. The line of action is the figure eight curve *P*, *A*, *C*, from which the tooth takes its name. This line of action is perpendicular to the tooth curves at the point *P* and tangent to the small circles *JK* and *LM*, which in turn are tangent to the great circle of which the tooth outlines are a part.

The cycloidal bevel gear tooth outline is formed by rolling a small circle of a sphere on a great circle. Fig. 221, *C* is the base of a spherical cone with vertex at *O* the center of the sphere. *SS* is a great circle of the same sphere and *PK* is the spherical epicycloid.

The spherical hypocycloid *PL* is formed by rolling the circle *D* on *SS* in the opposite direction.

The outlines of cycloidal bevel teeth are like those of spur gears except that the bevel teeth never have radial flanks, as no cone can be made to generate a plane surface by rolling inside another cone.

233. Making Bevel Gear Teeth.—Four methods are in use for making bevel gear teeth: (a) molding; (b) milling; (c) planing; and (d) the molding-planing process.

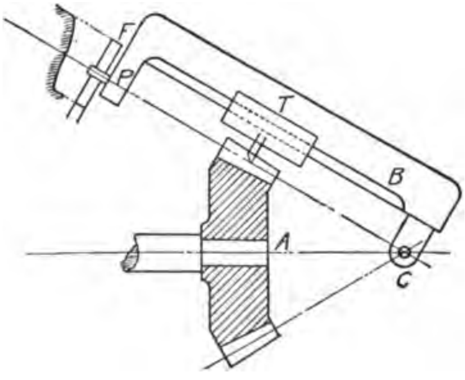


Fig. 222.

By the first method the teeth are molded from a pattern and cast in sand. Such teeth are suitable only for rough work, as the spaces must be cast wider than the teeth and there is always excessive backlash.

Bevel gear teeth may be formed by the milling process with rotary cutters. Since the spaces between the teeth are narrower toward the apex of the cone, a single cutter cannot be used to form the whole space at one cut. The teeth are made by going over the blank several times, the first cutter merely roughing out the stock, two finishing cuts being used one on each side of the space. Considerable practice is necessary on the part of the workman to adjust the cutters properly, accuracy being a matter of experience.

Fig. 222 illustrates the method of making the teeth by the planing process. *B* is a bracket mounted so as to swing freely on a pivot *C*. At the other end of the bracket a pin *P* is fixed so as to be held against the edge of the templet *F*. *T* is a tool holder which slides along the bracket.

The gear to be cut is mounted on a spindle at *A* and can be rotated or held firmly in a given position. The templet *F* is similar in form to the tooth to be cut.

The spaces are first roughed out, then the tool *T* is brought against the tooth and given a reciprocating motion, while at the same time it is advanced into the space by swinging *B* on its pivot. *P* guides the bracket so that the tooth outline is a duplicate of the templet in form but not in size. The tooth is formed by the linear planing process, each stroke of the tool being in the direction of the tooth elements.

This method is applicable to all forms of bevel gear teeth and is the most accurate general method in use.

The *octoidal* tooth and one other form, known as the *planoid*, may be formed by the molding-planing process which has already been described under spur gears. The cutting tool is made like the side of a crown gear tooth, Fig. 220, which is thus made the parent form for all teeth of the system. The direction of motion of the reciprocating tool is not parallel to the tooth elements but is given the direction of that element which is at the bottom corner of the tooth space.

234. Tredgold's Method.—The difficulty of laying out tooth curves on a spherical surface is apparent. An approximate method, first published by Tredgold, is commonly used whereby the reference surfaces of bevel gears are treated as cones of revolution tangent to the spherical surfaces of reference. In Fig. 223 the tooth outlines are described on the cone *QBC*, which is normal to the pitch cone *OBC* and has its elements tangent to the sphere on which the tooth curves actually lie. The pitch surface of the normal cone is assumed to be covered with some flexible substance which is unwrapped and laid out flat as at *DEF*. Teeth of any system may then be laid out on this plane surface, which is again wrapped around on the normal cone and the tooth outlines take their proper position.

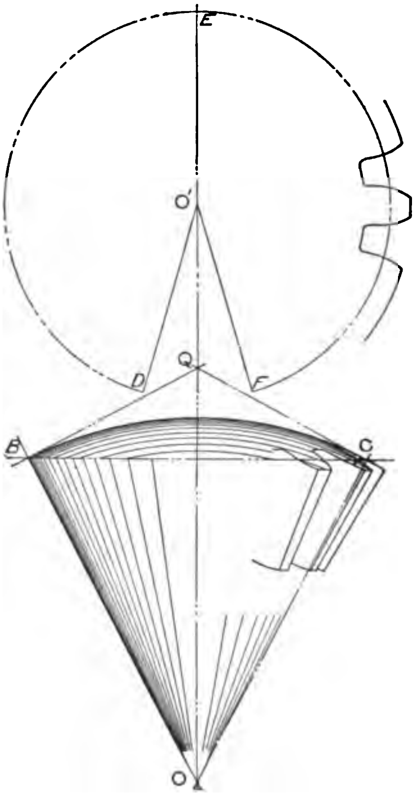


FIG. 223.

The error introduced by this method is too small to be appreciable unless the pitch is coarse and the pinion small in proportion to the gear.

235. To Lay Out a Pair of Bevel Gears.—Assume a velocity ratio of 7 to 6, a pitch of 3 and let the driver have 18 teeth and the follower 21 teeth. Involute system.

In Fig. 224, let *OA* be the driver and *OB* the follower axes, and let them intersect at *O*. Lay off on *OA*, 7 equal parts, and on *OB*, 6 equal parts. Draw *NK* and *MK* parallel to the axes and then draw *OK*, the common element of the pitch cones. Set off on the driver's axis a distance equal to the pitch radius of the follower, also set off on the follower's axis a distance equal to the radius of the driver. Through *A* and *B* draw the perpendiculars which intersect in *F*. These will be the pitch lines of the gears. Construct the pitch cones with common vertex at *O* and vertex of the normal cone of the driver at *Q*. With *OF* as a radius and any convenient point as a center draw the arc *FDG*, then the sector *QFDG* represents the development of the normal pitch cone, on which the teeth are laid out by Tredgold's method. Draw the pressure line, construct the base circle and draw the tooth curves as for spur gears. The teeth on this development will be shown in their true size and the arc *FDG* will equal the pitch circumference of the driver. Calculate the addendum and dedendum in the usual way and complete the teeth on the development. Lay off *GT* equal to the addendum and *GS* equal to the dedendum. Draw *ST* perpendicular to *OG*. The length of the tooth *GH* is independent of any kinematical principle and may be

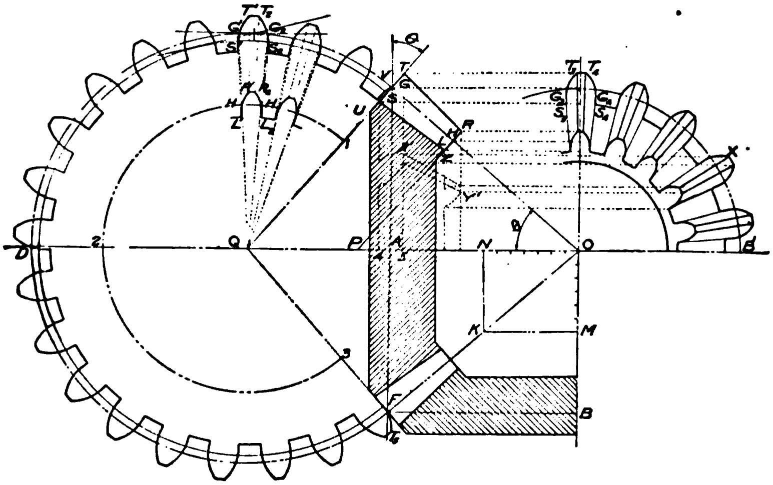


FIG. 224.

taken equal to one-third of GO. Erect a perpendicular to GO at H and draw TR and SL converging to the vertex O, then $STRL$ is a side elevation of one tooth. To complete the other views of a tooth, draw HP parallel to GQ, with PH as a radius draw the arc 1, 2, 3, concentric with FDG, this will be the pitch circle of the inner normal cone. With PL and PR as radii draw the addendum and dedendum lines of the inner ends of the teeth. From points on the developed tooth outline draw $S'Q$, $G'Q$, $T'Q$, etc., then the intersections L', H', R' will be points on the curve of the inner end of a tooth and $T'R'$, T_2R_2, will be elements of the developed tooth.

To draw a front view of the teeth take radius AG and draw an arc of the true pitch circle. Lay off the circular pitch and the tooth spacing. Project T, S, R and L upon a perpendicular through O and through these points draw circles to represent the top and bottom of both ends of the teeth. Lay off $T_3T_4 = T'T_2$, $G_3G_4 = G'G_2$ and so on, making the tooth outlines in the projection like those in the development except that they are foreshortened. All the other teeth on this projection are then made like the one in the vertical position.

To get the projection of any tooth in the side view, project points on the tooth outline from the front view. Thus X will be projected to X', Y at Y', etc.

The dimensions of the gear blank can be found when the pitch diameter and cone angles are known. Thus, the external diameter is equal to the pitch diameter plus twice the distance GV and $GV = GT \cos \theta$. θ can always be found from the known pitch radii and angle between the shafts.

236. Skew bevel gears, Fig. 225, may be used to transmit motion between a pair of axes which are neither parallel nor intersecting when the distance between the axes is small.

The pitch surfaces of such gears are hyperboloids of revolution, the gear blanks being frustra of the hyperboloids. The frustra taken may be either near to or remote from the gorge circles. If taken near to the gorge circles, the blanks should approximate closely to the hyperboloidal outlines, if remote the blanks will be practically like those of bevel gears, the pitch surfaces being cones tangent to the hyperboloidal outlines.

The surface of reference upon which the tooth outlines of a skew bevel gear would be correctly shown is a warped surface normal to the hyperboloidal pitch surface and the tooth outline would be formed by the intersection of all the tooth elements with this surface. This method of representing the tooth outlines is, however, impractical because it is impossible to obtain or imagine two surfaces of reference, generated on revolving askew axes, which can be made to coincide so that a pair of tooth outlines drawn upon them can be brought into contact. The teeth may, however, be laid out approximately, and studied on normal cones as in the case of Tredgold's method for bevel gears.

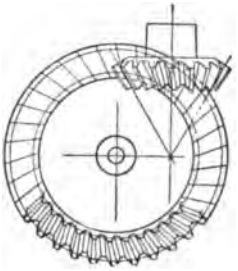


FIG. 225.

237. Teeth of Skew Bevel Gears.—The correct form of and method of generating skew bevel teeth on hyperboloidal pitch surfaces has not been clearly understood even by technical writers. It has been taught that correct skew bevel tooth surfaces, cycloidal in form, could be produced by rolling an auxiliary hyperboloid, externally and internally, on the pitch hyperboloids of a pair of mating gears, after the manner of generating spur teeth by rolling cylinders. MacCord has demonstrated that two tooth surfaces generated thus are not tangent along a common element of the two teeth and therefore that they will not work.

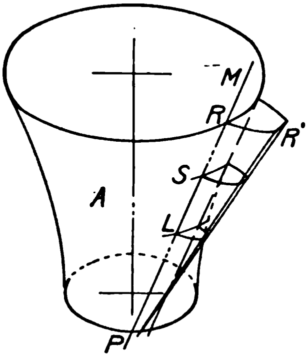


FIG. 226.

The best practical form of the tooth is undoubtedly that described by MacCord, in which the front of the tooth is a surface of single curvature and the back a warped surface, the back and front faces of the tooth section being involutes or conjugates of the involute.

In Fig. 226, let A be a hyperboloidal pitch surface with directrix PM. Pass a series of planes perpendicular to the axis of the hyperboloid and cutting its surface in circles parallel to the gorge circle. Now imagine PM to move in such a manner that, while remaining always in contact with the gorge circle, points on *PM* above the gorge circle, as *L*, *S* and *R*, describe involutes of the parallel circles. These involutes will be the outlines of sections of the front face of a tooth. The nearer the tooth sections are to the gorge circle the more flattened they will become and at the gorge circle the sides of a tooth will coincide with it. That is, the teeth at the gorge circle will have no sensible height and will disappear.

238. Contact Between Mating Teeth.—If a mating hyperboloid be placed in contact with *A*, along a common element parallel to *PM*, this second hyperboloid may have generated upon it a tooth surface similar to that of Fig. 226 by a movement of the generating line *PM*, around the second axis, identical with that already described. These two tooth surfaces, however, will not work together, for they will not be tangent along their elements. This is proved by the fact that a plane tangent to either tooth surface at a given point is determined by the element of the surface and the tangent to the involute at that point. But involutes of mating sections of the two hyperboloidal pitch surfaces do not lie in parallel planes and therefore the tangents to these involutes have different directions.

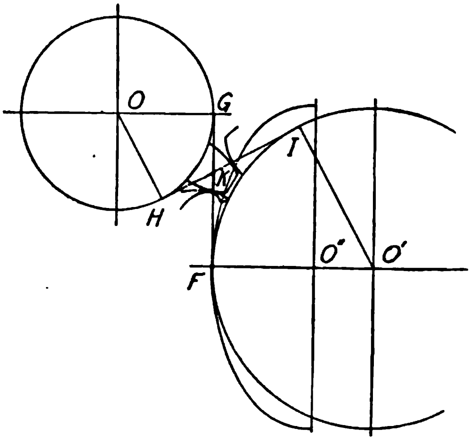


FIG. 227.

The failure of these tooth surfaces to work in tangent contact may be demonstrated by the aid of Fig. 227, in which the two base circles *O* and *O′* are supposed to have involute teeth. Let these base circles belong to mating sections of a pair of skew bevel gears, then their planes will make an angle equal to the angle between the axes of the hyperboloids and their intersection will be the line *FG*. If the two circles be now assumed to lie in the *same* plane, the line of action between their involutes will be *FG*. If the right-hand circle be rotated about *FG* as an axis until its center *O′* is projected at *O″*, it will take the actual position of the hyperboloidal section which mates with *O* and the line *FG* will still be the line of action, or the locus of contact between involutes of these circles, but for involutes of circles on either side of these planes, *FG* will not be the locus of contact nor will it be parallel to such other loci. Hence if these planes be considered as lamina of the pitch hyperboloids and given some thickness, corresponding to a pair of practical gears, the teeth will not work together in line contact. They may, however, be made to work in point contact by rounding off the teeth of one gear so that the elements are crowned lengthwise of the tooth. A better method of insuring contact is to make the fronts of the teeth of one gear true involute surfaces and then construct a conjugate tooth by a method similar to that of Art. 195. This conjugate will be a surface of single curvature.

The locus of contact for the backs of the teeth in Fig. 227 was the other tangent *HI*. This line will intersect *FG* at *K* and after *O′* is revolved to its new position, the two points on the backs of the teeth which originally met at *K* will still meet at that point, but no other points on the backs will come into contact at all, hence the teeth will not drive in the reverse direction.

The tangents to the involutes of the backs of the driver's teeth are not all parallel as was the case with fronts of the teeth, that is the backs of the teeth are warped surfaces. Teeth that will transmit motion in either direction may be made by constructing these warped surfaces and then constructing a conjugate surface on the back of the follower's tooth.

239. Velocity Ratio.—The angular velocities of a pair of skew bevel gears are to each other as the velocities of their hyperboloidal pitch surfaces, that is to say, the angular velocities are to each other in the inverse ratio, not of the perpendiculars let fall from any point of the common element upon the axes, but of the *projections* of those perpendiculars upon a plane parallel to both axes and the common element. This is equivalent to saying that their velocities are inversely proportional to the sines of their skew angles when the two gorge radii are proportional to the tangents of the skew angles.

When the teeth of skew bevel gears roll on each other, there is a simultaneous sliding motion along the elements of the teeth. Since the skew angles are not necessarily the same it follows that the two gears may have different circular pitches and that the number of teeth on the two gears will not be proportional to the diameters of their base circles. Therefore if the diameter of one of a pair of these gears is given for a given velocity ratio, that of the other must be found by construction.

240. Making Skew Bevel Gears.—The difficulty of even approximating the correct tooth surface without special tools for that purpose has practically prohibited the use of skew bevel gears and an extra bevel gear has generally been used to connect non-parallel axes.

In large sizes the gears may be produced by casting, but for small gears this method is too rough. They may be cut with a rotary cutter by mounting the gear blank on a milling machine in such manner that the cutter will follow the direction of the tooth elements, first one side of the tooth being finished and then the other, as in the similar process with bevel gears. This is a cut and try method which will require some skill in setting the blank at the proper angle. Where this method of cutting is used the blank is sometimes rotated simultaneously with the cutter, producing a spiral tooth. Such teeth will work in point contact with fair results. Either the straight or spiral tooth made by this process will generally need some filing before it will work at all.

Skew bevel teeth with straight-line elements are now produced by the linear planing process on automatic machines which use a former for guiding the tool as in cutting bevel gears, the only difference being that the position of the former is such that the cutting tool is guided along elements which are tangent to the gorge circle of the hyperboloidal pitch surface instead of meeting at the apex of a cone.

241. To Design a Pair of Skew Bevel Gears.—Let it be required to lay out a pair of 6-pitch skew bevel gears for a velocity ratio of $\frac{\text{Driver}}{\text{Follower}}=\frac{5}{4}$. The vertical projection of the axes on a plane parallel to both and to the common element is to make an angle of 45 degrees, the skew angle of the driver being 20 degrees and that of the follower 25 degrees. Let the driver have 24 teeth and its thickness be 2 in.

The circular pitch corresponding to a 6-pitch tooth is 0.5236 in. and the circular pitch of the driver on a plane perpendicular to its axis will be,

$$\frac{0.5236}{\cos 20^\circ}=0.558 \text{ in.}$$

Its pitch diameter will be,

$$\frac{24\times 0.558}{3.1416}=4.27 \text{ in.}$$

The circular pitch of the follower will be,

$$\frac{0.5236}{\cos 25^\circ}=0.5775 \text{ in.}$$

From the given velocity ratio the driver will have 30 teeth from which its diameter will be,

$$\frac{30\times 0.5775}{3.1416}=5.51 \text{ in.}$$

The pitch surfaces must now be constructed by the method of Art. 148. The diameter of the follower will be found by actual measurement of the diagram, to be approximately 5.25 in. Since this does not agree with the diameter as calculated from the number of teeth the skew angles may be changed and the construction modified until the diameters found by the two methods do agree.

The teeth must now be laid out on the back of the normal cones as in Tredgold's approximation. The normal cones, Fig. 228, are found by constructing tangent cones to the pitch surfaces at the middle of the frustra, *K*, and then drawing the sides of the normal cone perpendicular to the sides of the tangent cone.

A complete tooth of the driver is shown at *L*. The outlines of the tooth are involutes constructed in the usual way, both ends of the tooth are similar, both are symmetrical with a radial line through the center of the section and their relative positions is determined by the skew angle of the gear. The acting face of the follower's tooth is conjugate to the tooth L and may be found as follows:

Pass any plane tangent to the acting face of L and perpendicular to its line of action. This plane will intersect the gorge plane of the driver in a line tangent to the gorge circle and as L rotates the plane will be pushed outward turning about its gorge plane trace as on a hinge. This turning plane will cut a section of the driver, taken perpendicular to the axis QQ, in lines to which the section

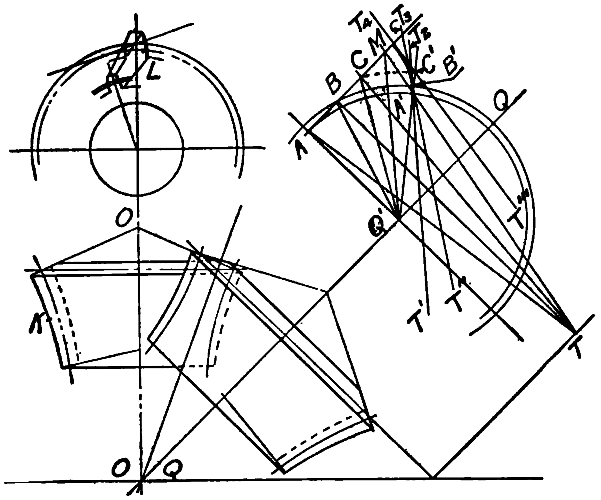


FIG. 228.

of the follower's tooth in that plane will be tangent. Let the point M in the line AM, trace an involute of the driver as the line is wound upon the base circle. At B, the intersection of AM with the pitch circle, draw a perpendicular intersecting the plane of the gorge at T. Then T will be the intersection of the gorge plane trace of the turning plane, with the plane perpendicular to QQ. Draw lines from T and from Q' to A, B and C. At A', the base of the involute, lay off the angle $Q'A'T'$ equal to $Q'AT$. The conjugate tooth at this point will be tangent to the line

$T'T_2$. In like manner lay off $Q'AT$ at the points B' and C' and finally draw the tooth curve tangent to the lines $T'T_2$, $T''T_3$, and $T'''T_4$.

The backs of the driver's teeth may be constructed by drawing three or more involutes and moving an element in contact with them from root to point, forming a warped surface. The conjugate back for the follower is constructed as follows: Pass a plane perpendicular to QQ and draw upon this plane the section cut by it from the back of the driver's tooth, rotate driver and follower simultaneously through a very small angle and draw another section cut from the driver's tooth, repeat this process until a sufficient number of curves have been determined and then draw the outline of the back of the follower's tooth tangent to these curves. Select another transverse plane and perform the same series of operations. When three or more such tooth outlines have been determined a right line sliding in contact with them from root to point will generate the tooth surface.

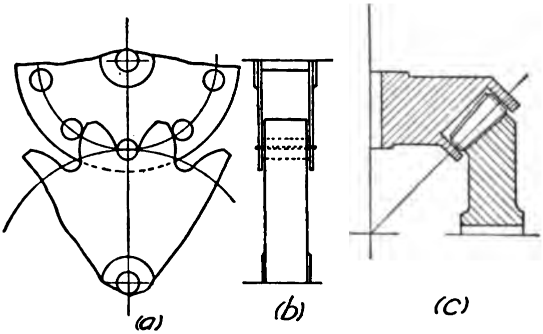


FIG. 229.

The process of drawing these tooth outlines is long and tedious and there is no simple way of showing the tooth surface on paper. They may be constructed by drawing the involute and conjugate outlines as described and using them as templets, each set in its proper relative position, as determined by the skew angle and then filling in the material of the tooth between and fairing it down to a smooth surface.

242. Pin Gears.—Fig. 229 illustrates three different forms of the pin gear.

In (*a*) the pins are set in the face of a flat disk, their centers being located on the pitch circle with their center distances equal to the circular pitch.

Fig. 229 (*b*), is called a lantern wheel. The pins are here supported by two disks and each pin has a roller with which the teeth of the mating wheel make contact.

Fig. 229 (*c*) is a bevel pin tooth. In this gear also the pin is supported at both ends and generally carries a roller.

Pin gears find their most common application in small work like clock trains but they may also be used successfully for heavy work. In the past they have been much used in wooden construction.

Pin gears are easy to make and they give a comparatively smooth contact, particularly when used with a roller. In small sizes the pin tooth with roller gives extremely little friction.

As generally made, pin gears are defective in their contact and in coarse pitches when the pin wheel is small in diameter they will not work properly without some modification of the theoretically correct form.

243. Construction of the Pin and Its Conjugate Tooth.—The circular pin is a true tooth curve of the segmental form, for that portion of its circumference which lies between its pitch circle and the tangent to the pitch circle at the center of the pin.

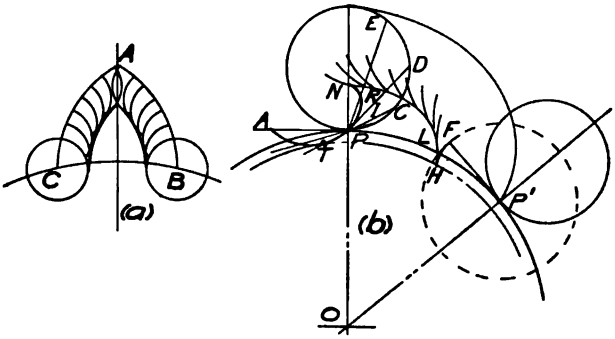


Fig. 230.

The general method of designing teeth for pin gears is to assume a diameter for the pin and then find a conjugate tooth curve to work with it.

The derivation of the pin tooth from the hypocycloid was explained in Art. 212 (*c*). In Fig. 230 (*a*), *AB* and *AC* are epicycloids generated by rolling the pitch circle of the pin wheel on that of the gear. The width of the tooth *CB*, is equal to the circular pitch and the pin is merely a mathematical point. Since a practical pin must have some sensible diameter the tooth is modified as follows: From points on the epicycloids *AB* and *AC*, and with radii equal to that of the required pin, draw a series of arcs and then draw the new tooth outline tangent to the arcs.

The sides of this new tooth outline are epicycloids parallel to *AB* and *AC*. These parallel epicycloids possess the peculiar property of being composed of two branches, *NH* and *HF*, Fig. 230 (*b*), with a cusp at *H*. This formation of the parallel epicycloid will cause interference if the teeth are coarse although it may be disregarded for fine teeth.

244. Contact between Pin and Tooth.—The line of action, or locus of contact, for the pin and its conjugate is a portion of the limacon and may be drawn as follows when the dimensions of the pin and tooth are known. In Fig. 231 let *P*, *C*, *D* and *E* be successive positions of the center of the pin on a given pitch circle. Draw chords of the pitch circle through these points and *P*, and produce them if necessary. On *EP* set off *ER*, equal to the radius of the pin, in the same way set off *DI*, *CT* and *PA*. A curve drawn through the points *A*, *T*, *P*, *I* and *R* will be a portion of the required limacon. The complete curve is shown in Fig. 230 (*b*) where it is drawn with a pin radius larger than that of its pitch circle so as to show up the cusp on the parallel epicycloid.

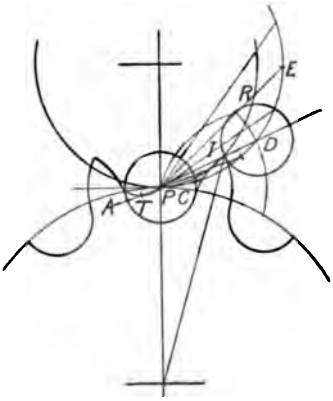


FIG. 231.

Contact between the pin and tooth is practically confined to one side of the line of centers and when the tooth is driving, as shown, it will be on the arc of recess. Since tooth contact is always smoother on the arc of recess it follows that the tooth should be made to drive the pin. If, however, the pin carries a roller the receding action is no better than the approaching action and the pin may be made the driver.

In Fig. 230 (*b*), the branch *HF* is inoperative as a tooth curve, and the construction of the branch *HN* is such that it cuts the circumference of the pin at some point *L* when the center of the pin is at *P′*. If the tooth curve is stopped off at *L* there will be an angle in the tooth outline which will in most cases cause a break in the velocity ratio. On the other hand, if the tooth curve be continued to the cusp at *H*, the portion of the tooth curve below *L*, will interfere with the pin. This interference may be avoided by locating the centers of the pins inside their pitch circle a distance such that their circumferences will pass through the point *H* instead of cutting the curve as at *L*.

The cusp is easily located for a given case for it will always lie on a circle drawn from the center *O* and tangent to the limacon at *T*. The locus of the centers of the pins may then be found by trial.

It is sometimes stated that contact begins when the center of the pin is at *P* and this would be true if the pin were a mere point corresponding to its center, but owing to the double branch of the derived curve and the methods of correcting for its effect, the point of contact is variable and may even be such as to give a negative arc of approach.

245. Limiting Size of the Pin.—A common method of designing pin gears is to make the diameter of the pin equal the thickness of the tooth at the pitch line but it is sometimes necessary to determine the limiting size of the pin for a given pitch and arc of action when the diameter of the pitch circles are known.

In Fig. 232, lay off *PA* and *PB* equal to the given arc of action or recess and from *B* lay off *BC* equal to the circular pitch. Bisect *BC* at *D* and draw the radial line *OD*. The end of any pointed tooth in this position and having the circular pitch *BC*, will lie on the line *OD* produced. The point of contact between a pin and tooth is always located on the line *PA* drawn from *P* to the center of the pin, because such a line is the common normal at the point of contact. The end of a pointed tooth is the last possible point of contact for that tooth on the arc of recess. Hence it follows that the last point of contact with a given pitch and arc of action will lie at *T* the intersection of the lines *OD* and *PA*. *AT* will then be the radius of the largest possible pin, for if it be increased its circumference will cut *AP* beyond *T* and the arc of recess will be shortened.

If the tooth be cut off on a line *NN* so as to have some width at the top, the arc of recess will again be shortened. If, on the other hand, the arc of recess and the height of the tooth be kept the same and the tooth be widened at the top as at *K*, then the diameter of the pin will be decreased.

PQ is the limiting length of the arc of recess for the assumed pitch, *Q* being the intersection of the point of the theoretical tooth that works with a pin of zero diameter, with the pitch circle of the pin wheel.

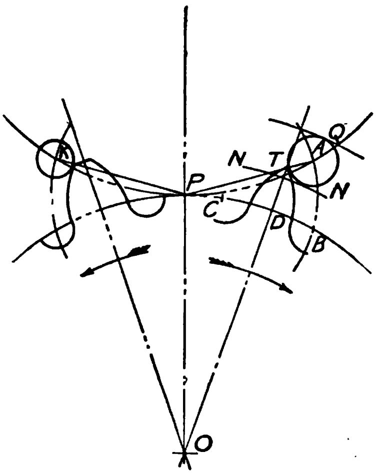


FIG. 232.

246. Annular pin Gears and Pin Racks.—Figs. 233 and 234 show four possible arrangements of annular pin gears and pin racks. In each case the tooth should drive the pin and the driving face of the tooth should be a curve parallel to the true curve, described in Art. 243, and at a distance from it equal to the radius of the pin.

In Fig. 233 (*a*), the tooth outlines are curves parallel to the hypocycloid and at (*b*) the tooth curves, which are on the small wheel, are parallel epicycloids.

In Fig. 234 (a), the rack teeth are parallels to the cycloid, while at (b) the wheel teeth are involutes of their own pitch circle generated by rolling upon it the pitch line of the rack.

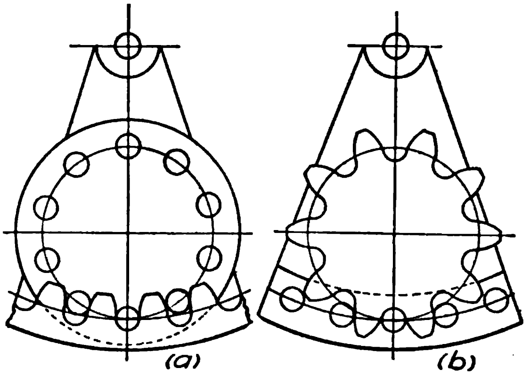


FIG. 233.

247. To Design a Pin Gear.—Let it be required to lay out a 7-pitch gear of 42 teeth to work with a pin wheel of 14 pins.

The diameter of the gear will be 42÷7 = 6 in., and the diameter of the pin wheel 14÷7=2 in. The circular pitch will be 3.1461÷7=0.448 in.

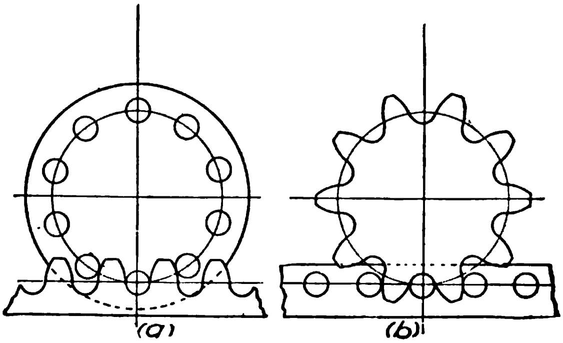


FIG. 234.

Draw the pitch circles and space off the centers of the pins. Assuming the teeth to be of standard length as given by Art. 197

the addendum will be $\frac{1}{7}$ in. Then draw the addendum line of the gear with a radius of $3\frac{1}{7}$ in.

Space off the teeth on the gear and describe epicycloids through each division point by rolling the pitch circle of the pin wheel on the pitch circle of the gear.

Assume that the tooth and pin are to have the same thickness, that is, each equal to one-half of the circular pitch or 0.224 in. With this diameter draw in the pins and then from centers on the epicycloids already found and with radius equal to one-half the diameter of the pins draw the derived curve as explained in Art. 243. Finally with centers on the pitch line of the gear and with radius equal to that of the pin draw the semi-circular flanks of the teeth to meet the derived curves at the pitch line.

248. Chain Gears.—These are of two forms according as they are used for (1) hoisting or (2) transmission.

There are also two types, kinematically, of class (1). The simple or direct hoist, like the marine windlass, for example, where the velocity of the lifted weight is equal to the pitch velocity of the wheel, and the differential hoist on which the velocity of the lifted load equals the difference in the linear velocity of two-pitch lines of different diameter but turning together on a single wheel with the same angular velocity.

The transmission chain gear, form (2), or the sprocket and chain, is used to transmit positive motion between axes that are too far apart to be connected by gears in direct contact, as in the driving mechanism of the bicycle or automobile.

The chain drive has certain practical defects which sometimes make its use unsatisfactory, particularly at high speeds. Among these are the flapping and jumping of the chain when in motion and the large number of joints which by wearing, cause the chain to stretch so that it does not fit the sprocket wheels.

The sprocket and chain has been used for speeds as high as 2000 ft. per minute but it is likely to be much more satisfactory at lower speeds.

249. The Hoisting Chain Wheel.—This type of chain and wheel is shown in Fig. 235. The wheel is so designed that every second link of the chain will fall into a pocket and be firmly held between two adjacent lugs, the alternate links lying edgewise in a groove on the median plane of the wheel. The circular pitch is the distance between the lugs and this is equal to the combined length of two links.

The diameter of the wheel depends upon the size of the chain and should be great enough to allow the latter to make contact with over half the number of lugs without being sharply bent. The number of lugs must be determined experimentally for a given size of chain and when this number is known the wheel may be designed.

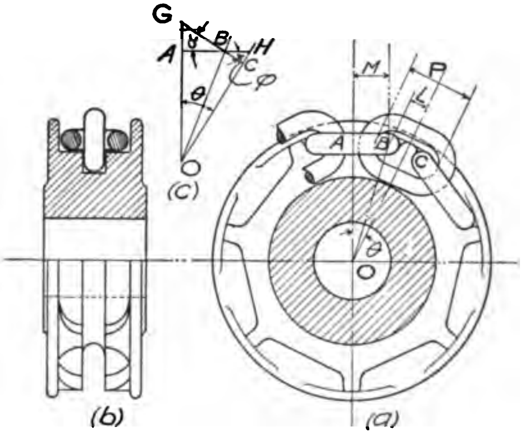


FIG. 235.

Let P be the inside length of a link,
D the diameter of wire in the chain,
N the number of lugs,
θ be one-half the angle subtended at the center of the wheel by the pitch arc.

Then

$$\theta = \frac{360^\circ}{2N}$$

and from the sketch (*a*),

$$M = \frac{P+D}{2} \text{ and } L = \frac{P-D}{2}$$

and both of these values are known from the size of the link and wire.

In the sketch (*c*) let the symbols mean the same as in (*a*) where AB is equal to M and BC equals L. Extend the lines OC and AB, until they meet at H, and produce CB to meet OA at G.

In the triangle BHC, angle $\varphi=\theta$.

By trigonometry, $BH=BC$ sec. $\varphi=BC$ sec. θ.

In the triangle ABG, the angle $\alpha=\theta$ and $BG=AB$ sec. θ.

Again $OA=(AB+BH)$ cot θ, in which by substituting the value of BH we have,

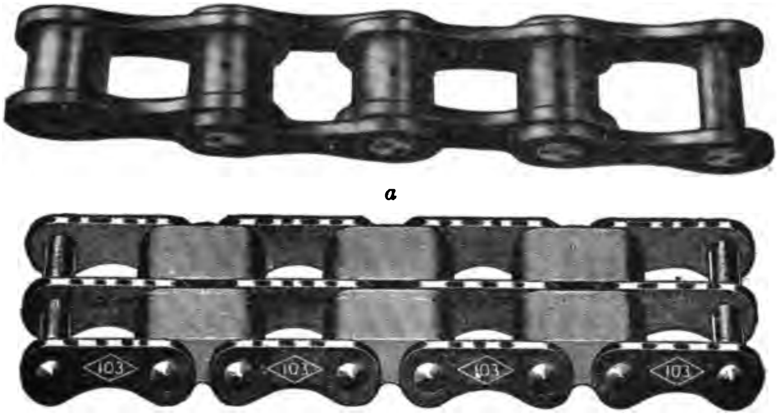
$$OA=(AB+BC \text{ sec. } \theta) \text{ cot } \theta. \tag{1}$$

By a similar process we may derive the equality,

$$OC=(BC+BG) \text{ cot } \theta=(BC+AB \text{ sec. } \theta) \text{ cot } \theta. \tag{2}$$

Now from the figure,

$$OB=\sqrt{OA^2+AB^2} \quad \text{or} \quad \sqrt{OC^2+BC^2}. \tag{3}$$



a

b

Fig. 236.

Using values of OA and OC given by (1) and (2) and values of AB and BC from the known lengths of the links, and solving equation (3) for OB we get the pitch radius of the wheel.

250. The Sprocket and Chain.—Fig. 236 shows two forms of sprocket chain, (*a*) the roller and (*b*) the block type. The pitch of the roller chain is the distance from center to center of the rollers and in the block chain it is the distance between either the middle of the blocks or the side links. The sprocket wheel for the block chain has fewer teeth than for the roller chain on account of the lengthened pitch.

The velocity ratio transmitted is constant and inversely proportional to the diameters or the number of teeth on the driver and follower. An indefinite number of pairs of sprocket wheels can be found for a given velocity ratio but a wheel will not work well if it has less than nine teeth.

In Fig. 237, if N be the number of teeth on the wheel, P the length of a link, and α the angle whose chord is P, then $\alpha = 360 \div N$ and the pitch diameter $= P \div \sin. \frac{\alpha}{2}$

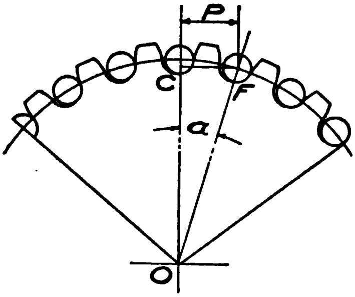


FIG. 237.

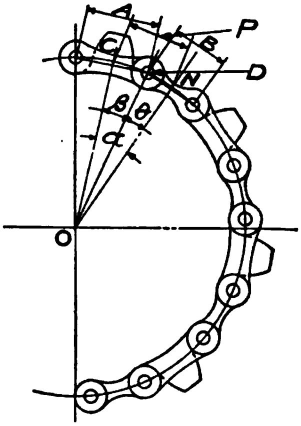


FIG. 238.

For a block chain, Fig. 238, let

N be the number of teeth as before,

A the length of the side links = 0.6 pitch generally,

B the length of the blocks = 0.4 pitch generally,

R Pitch radius of wheel,

β the angle whose chord is $\frac{A}{2}$,

θ the angle whose chord is $\frac{B}{2}$,

and $\alpha = \beta + \theta$

α also equals $\frac{360}{2N}$.

In the right triangle OCD,

$$\sin\beta=\frac{CD}{OD}=\frac{A}{2R}. \qquad (1)$$

In the right triangle ODN,

$$\sin\theta=\frac{DN}{OD}=\frac{B}{2R}$$

but $$\theta=(\alpha-\beta)$$

hence $$\sin(\alpha-\beta)=\frac{B}{2R}.$$

Expanding $\sin(\alpha-\beta)$, we get,

$$\sin\alpha\cos\beta-\cos\alpha\sin\beta=\frac{B}{2R}.$$

Substituting in this equation the value of $\sin\beta$, we have

$$\sin\alpha\cos\beta-\frac{A}{2R}\cos\alpha=\frac{B}{2R},$$

from which,

$$\sin\alpha\cos\beta=\frac{A\cos\alpha+B}{2R}$$

and $$\cos\beta=\frac{A\cos\alpha+B}{2R\sin\alpha}. \qquad (2)$$

Dividing equation (1) by (2), we have,

$$\tan\beta=\frac{\frac{A}{2R}}{\frac{A\cos\alpha+B}{2R\sin\alpha}}=\frac{A\sin\alpha}{A\cos\alpha+B}=\frac{\sin\alpha}{\frac{B}{A}+\cos\alpha} \qquad (3)$$

After finding the value of β from (3), the pitch diameter, which equals $\frac{A}{\sin\beta}$, may be determined.

In both the roller and block type of sprockets the centers of the pins are on the pitch line when the chains are in place on the wheel.

The outlines of the teeth above the pitch line, are arcs of circles drawn from the centers of the rollers, in case of the roller chain and from the centers of the pins in the side links in the case of the block chain. The bottoms of the spaces of the roller sprocket are arcs of circles drawn from centers on the pitch line and with a radius equal to that of the roller. In the block chain sprocket, the bottoms of the spaces are formed by joining these arcs by another arc drawn from the center of the sprocket wheel.

The width of the teeth should be made slightly less than the diameter of the rollers to allow for clearance, and their length should be such that they will extend somewhat outside of the chain.

The thickness of the teeth is arbitrary. They should be made somewhat thinner at the top than at the pitch line so that they will enter the chain when in motion without hitting against the side links.

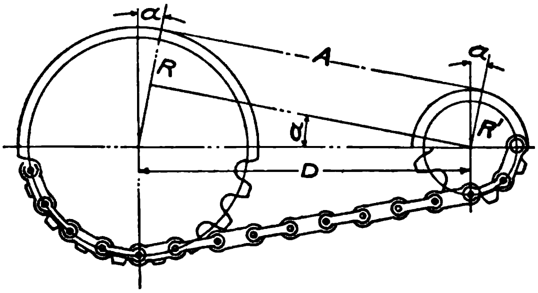


FIG. 239.

251. Length of the Chain.—In Fig. 239,

Let D = the distance between the centers of the sprocket wheels,
A = the distance between the points of contact,
R = the pitch radius of the large sprocket.
R' = the pitch radius of the small sprocket.
N = the number of teeth on the large sprocket.
N' = the number of teeth on the small sprocket.
P = the circular pitch.

From the figure,

$$\alpha = \sin^{-1}\frac{R-R'}{D},$$

and

$$A = D\cos\alpha.$$

Then the length of the chain will be,

$$L=\frac{180+2\alpha}{360}NP+\frac{180-2\alpha}{360}N'P+2D\cos\alpha. \tag{1}$$

The unsupported distance between the sprocket wheels should not exceed 10 or 12 ft.; on longer drives the chain may be supported by sprocket idlers.

A chain will run better with some slack and one of the sprocket wheels should be mounted on a sliding support so that the length of the chain may be adjusted.

The chain drive will give the best service when the plane of the chain and sprockets is vertical and thc axes at the same level.

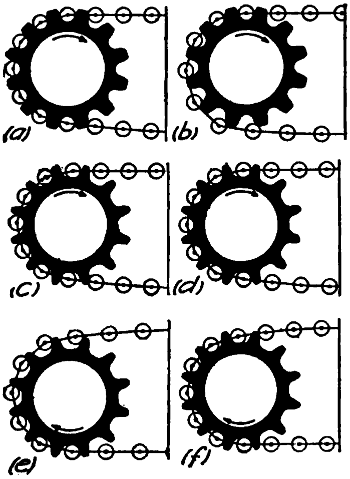


Fig. 240.

252. Interference between Sprocket and Chain.—Fig. 240 shows how chains climb the sprocket after they are become worn so that the pitches do not correspond. (*a*) is a driven wheel with chain in good condition and (*b*) is the same wheel after wearing and stretching of the chain. If a sprocket is cut with some clearance between the teeth and roller, as in (*c*), when the chain stretches it will take the position shown by (*d*) and the action will be practically as good as new. At (*f*) and (*e*) are shown driving sprockets, cut with clearance, before and after stretching the chain. Theoretically, each block or roller on the sprocket takes its share of the load, but in practice this is never true, all of the load being thrown on the first one or two links on the driving side. If the pitch of the chain is made longer than that of the sprocket the chain tends to fit, when new, as shown at (*e*), the rollers will work backward as wear continues, but the chain will work well until the condition indicated at (*f*) is reached. This increasing of the pitch will add to the life of the chain and may be adopted when the driving is to take place in only one direction.

253. Adjustable Link Chains.—These are sometimes used in place of belts for heavy driving. At speeds below 2000 ft. a minute they may be made to run with very little noise and high efficiency. The losses are due to the internal friction of the chain and to its frictional contact with the teeth of the wheel.

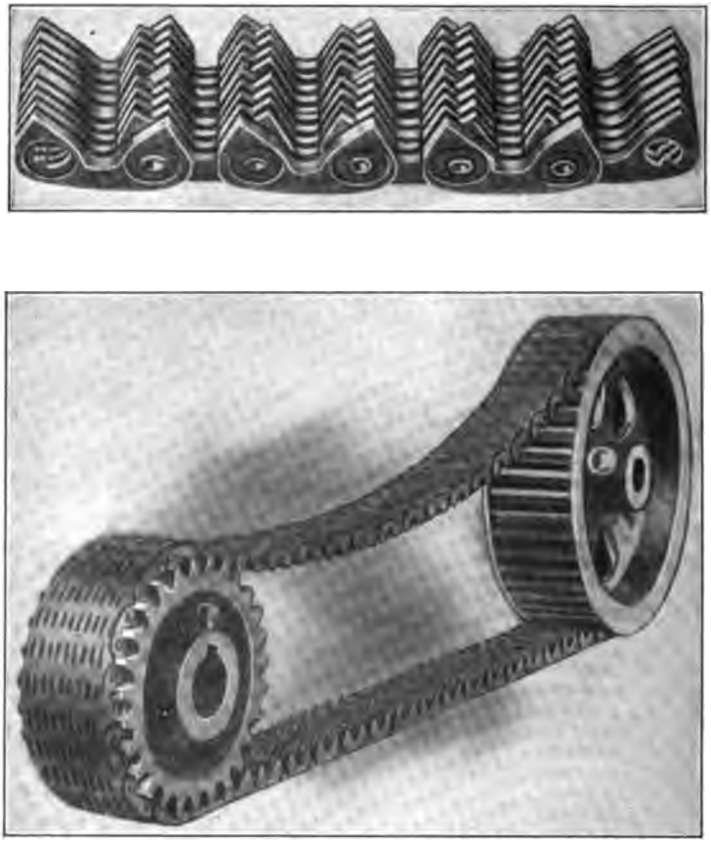


Fig. 241.

Two types of chain may be noted. Fig. 241, known as the Morse chain, has a link mounted on a hardened steel roller made in two parts, and as the chain bends to the form of the sprocket wheel the two parts of the pin roll on each other through a small angle, allowing the chain to bend with very little internal friction.

The Renold chain, Fig. 242, has the link designed to eliminate the maladjustment of the chain on the sprocket wheel after wearing and stretching has taken place. The peculiar form of the link allows it to seat itself firmly on the pulling side of the wheel and to remain in that position without slipping until it is released on the outgoing side.

Fig. 242 (*a*) shows the contact when the chain is new and (*b*) shows the same after it has become worn.

254. Design of a Sprocket and Chain.—Let it be required to design a sprocket and chain drive for a center distance of 5 ft., velocity ratio of driver to follower to be 3 to 1, driver to have 12 teeth and circular pitch to be 1½ in.

If a block chain be assumed, the distance, Fig. 238, *A*, for this pitch, will be 0.9 in. and *B* will be 0.6 in.

From Fig. 239,

$$\alpha = 360 \div 2N = 360 \div 24 = 15 \text{ degrees}$$

Since from Art. 250, the relative proportions of the link and block are 0.6 and 0.4, $\beta = 0.6 \times 15$ degrees $=$ 9 degrees and $\theta = 0.4 \times 15$ degrees $=$ 6 degrees.

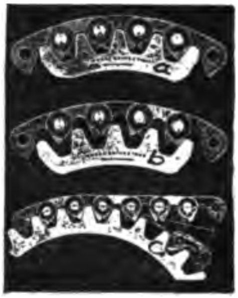


FIG. 242.

Then the pitch diameter of the driver equals $A \div \sin 9$ degrees $= 5.76$ in., and that of the follower equals $3 \times 5.76 = 17.28$ in.

The length of the chain for a center distance of 5 ft. will be, from formula (1) of Art. 137:

$$L = \frac{180+30}{360} \times 36 \times 0.15 + \frac{180-30}{360} \times 12 \times 1.5 + 120 \times 0.997 =$$

154.92 in. or 12.9 ft.

255. Face Gears.—Two pin wheels may be used to transmit motion between axes that are not parallel, such an arrangement being called a face gear, Fig. 243. This is an old form, commonly constructed of wood, which was used to considerable extent before the introduction of bevel and skew bevel gears.

It has the advantage of being easy to construct, as both the pins and the plates can be turned in the lathe. The pins bear on each other in point contact, therefore they are under the disadvantage of transmitting all of the pressure through a single point.

As constructed, these gears were generally made in the form of two disks, exactly the same size and upon axes at right angles, either in the same or in parallel planes. They can, however, be made to work when the wheels are of different sizes and when the axes are not at right angles.

If the axes are not askew the velocity ratio will vary inversely as the diameter of the pin circles.

The directional relations will be the same as for the corresponding case of bevel gears.

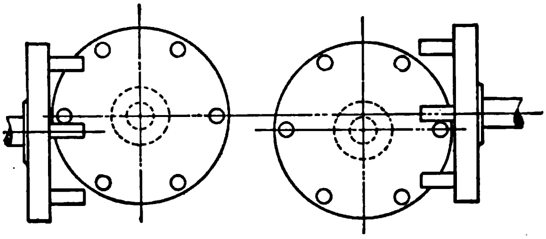


Fig. 243.

256. Shape of the Pins.—The circular pitch only, is used in connection with face wheels and this is the distance between the centers of the pins measured on the pitch circle passing through their centers. The shape of the pins will be determined by the normal distance between the axes and by the relative size of the pitch circles.

The pins on one of the wheels may always be made cylindrical and from this the shape of the conjugate pin may be derived.

The following variations will affect the shape of the derived pin:

Case I.—Axes in the same plane. (a) Driver and follower equal in diameter. (b) Driver and follower unequal in diameter.

Case II.—Axes in parallel planes. (a) Driver and follower equal in diameter. (b) Driver and follower unequal in diameter.

257. Axes in the Same Plane.—Case I, (a) is illustrated in Fig. 244. Two pins, in contact at P on the line of centers, would be mere lines corresponding to their axes. The correct form and size for practical pins may be determined thus:

Assume the upper wheel to have cylindrical pins of sufficient size to meet the required conditions. Place the wheels so that the center of the assumed pin is at *P* and in contact with the center line of its conjugate pin N. From this position, rotate the lower wheel until the distance PN equals the radius of the cylindrical pin plus the desired radius at the base of the conjugate pin. Ordinarily this distance will be equal to the diameter of the assumed pin. If both wheels be now rotated simultaneously in the direction shown the distance between the centers of the pins will become less as they recede from the line of centers and when they reach the positions G and G' their center distance will be

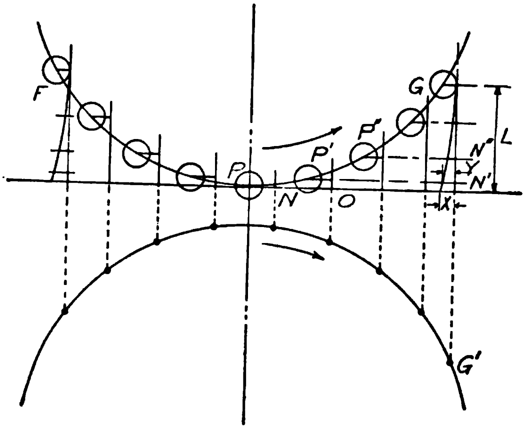


Fig. 244.

equal to the radius of the assumed pin and the conjugate pin will come to a point. Divide the distance L into any convenient number of parts and the arc $P\,G$ into the same number of parts. Draw the horizontals $P'N'$ and $P''N''$, etc., through the division points, and set off x equal to the distance PN, minus the radius of the pin and y, equal to $P'O$, minus the radius of the pin, and so on. A curve drawn through the points so found will be the required outline of the conjugate pin. Ordinarily this derived pin will give satisfactory contact, but if the pins be relatively large and the pitch circles small then points on the derived pin should be found by taking the distances from its center to the point of contact with the cylindrical pin, along their common normal instead of on the parallels shown. In the figure the pitch is greater than the sum of the diameters of the two pins at their bases, the conjugate pin is constructed from the arc of recess and the contact takes place on that arc. This may be seen from the fact that the centers of the pins are further apart on the arc of approach. If the sum of these pin diameters be made equal to the pitch arc, then contact will take place during both the approaching and receding action.

In Case 1 (*b*), where the wheels are not the same diameter, the shape of the derived pins will be as shown in Fig. 245, *x* and *y*, the smaller of which will be used.

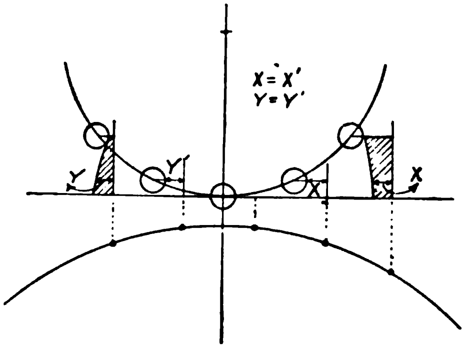


FIG. 245.

If the cylindrical pins be given to the smaller wheel, it must be made the follower if contact is to take place on the arc of recess. If the cylindrical pins be placed on the larger wheel the derived pin will be similar in form to the one shown at *y*, but not so long, and if contact is to take place on the arc of recess the large wheel must still be the driver.

258. Axes in Parallel Planes.—Case II, (*a*) in Fig. 246 is shown a pair of face gears where both wheels have the same diameter and both have cylindrical pins. *L*, the perpendicular distance between the axes, is equal to the diameter of a pin.

Either wheel may be made the driver and the contact will take place on both the arcs of approach and recess.

For any position of two contact pins the perpendiculars *QR* and *TS* are equal. The radii of the wheels being equal, the angles whose sines are *QR*÷*Rad.* and *TS*÷*Rad.* are equal and hence the angular velocities of the wheels are equal.

The size and pitch of the pins must be so selected that *M* on the upper wheel will not strike against the top of *N* in approaching the line of centers.

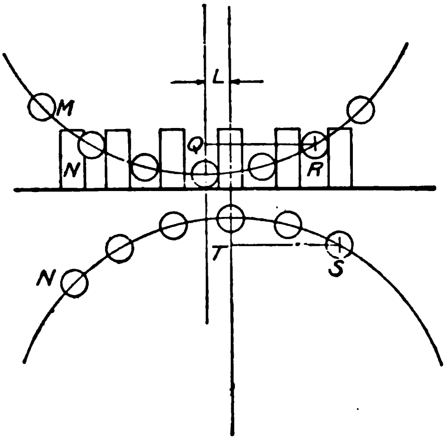


Fig. 246.

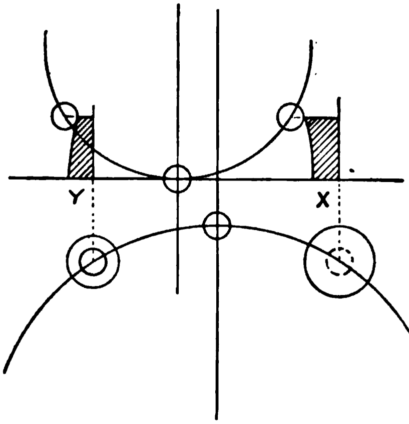


Fig. 247.

When the distance between the axes, *L*, is not equal to the pin diameter, one of the wheels may have cylindrical pins and the other conjugate pins derived from it by the method of Art. 243.

Fig. 247 illustrates Case II (*b*), where the diameters of the wheels are unequal. The cylindrical pins may here be placed on either wheel, but the larger wheel will always be the driver if contact is on the arc of recess. The shape of the pins for contact on the arc of recess is shown at *Y*, and their shape if contact is to be on the arc of approach, at *X*. Pins like that at *X* would work with contact on the arc of approach, but after passing the line of centers they would interfere with the cylindrical pins of the follower. With the cylindrical pins on the large wheel, or driver, *Y* would still be the correct pin for contact on the arc of recess and *X* would cause interference on that arc.

259. Irregular Gears.—A classification that would include all irregular gears would be difficult to make because of their great variety. Such gears are generally used to transmit either (a) an intermittent motion, (b) a variable velocity or (c) both of these together.

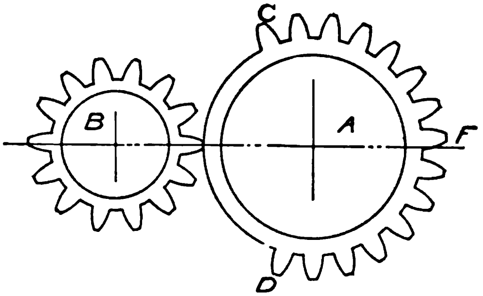


FIG. 248.

In Fig. 248 is illustrated one form of an intermittent gear. While *A* turns through the arc *CFD* it will drive *B* through as many revolutions as the whole number of the teeth on *B* are contained times in the teeth on the arc *CFD*. While *A* turns through the arc *CD*, the wheel *B* will remain at rest.

A fault of this construction is the tendency of the wheels to jam when the teeth go into action. The jamming can be avoided by making the wheels as shown in Fig. 249, *A* being the driver, no motion is transmitted to *B*, while *A* is turning through the arc *CFD*. A portion of *B* is cut away so that it is locked from motion in either direction until the pin at *C* engages the catch, then the teeth are thrown into contact, and while *A* turns through the arc *CD* it drives *B*. By regulating the number of teeth on the wheels, the amount of motion transmitted can be varied at pleasure.

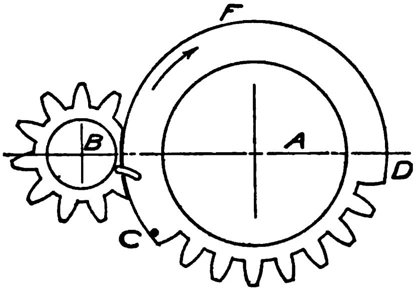


FIG. 249.

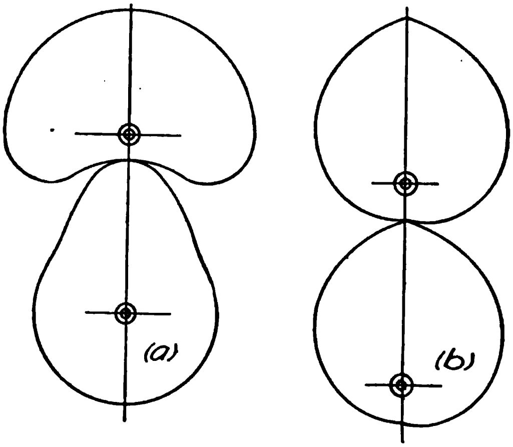


FIG. 250.

260. Gears for Variable Velocity Ratio.—These may be divided into three classes: (a) Dissimilar pitch lines; (b) unsymmetrical pitch lines; (c) symmetrical non-circular pitch lines.

An example of class (a) is shown by Fig. 250 (*a*), which may represent any two curves capable of acting together in rolling contact. At Fig. 250 (*b*) is shown a pair of unsymmetrical pitch lines. Such pitch lines are commonly made up by putting together one or more regular curves as the logarithmic spiral or the elliptical arc. In all gears having non-circular pitch lines the teeth should be placed so that the center line of each tooth is perpendicular to the tangent to the pitch line at that point. With such an arrangement of the teeth, the obliquity of action will be greater at certain parts of the contact than at others and the pitch lines should be so selected that its maximum value will not exceed from 40 to 50 degrees.

Mating gears on symmetrical pitch lines, whether similar or dissimilar in form, may be made to work together. If teeth be given to any of the lobed wheels, of Arts. 157 to 161, they will belong in the class of symmetrical, non-circular gears.

261. Elliptical Gears.—Whenever it is required to produce a simple variation in speed between fixed limits, the velocity ratio between those limits being unimportant, a pair of elliptical gears may be used with pitch surfaces that are either elliptical cylinders or cones, according to the relative position of the axes.

Such gears furnish a convenient method of producing the quick return motions used on metal-cutting machines of the reciprocating type, which have a slow powerful motion on the forward, or working, stroke followed by a rapid motion on the return stroke.

The teeth of elliptical gears may have any form that is applicable to spur gears, the involute being most generally the one used. Theoretically, each tooth outline varies with its position on the gear, no two consecutive teeth being alike. The labor and expense of making so many different teeth would be so great that it is common to approximate the elliptical pitch line by four circular arcs and construct all of the teeth on each of these arcs as for spur gears.

262. Contact and Velocity Ratio.—Elliptical gears are made for external contact only, hence the gears rotate in opposite directions.

The contact and velocity ratio depend upon whether the center of rotation is taken at:

(a) The foci, or
(b) The center of the ellipse.

Fig. 251 illustrates the case where each ellipse rotates about one of the foci as a fixed center. The velocity ratio varies between the limits $\frac{OP}{O'P'}$ and $\frac{OQ}{O'Q'}$ and has one maximum and one minimum value in each revolution.

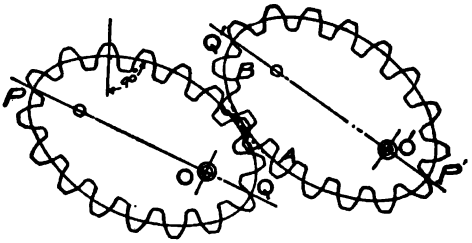


Fig. 251.

The contact is mixed rolling and sliding, the locus of contact being along a line which varies with the curvature of the ellipse. For an involute tooth this is not straight, as in the case of spur gears, its form must be determined by trial for each gear.

The driving action is positive during that part of the revolution in which the contact radii of the driver are increasing in length. During the balance of the revolution the teeth may be held in contact by means of a link as shown by Fig. 49.

If the center of rotation is taken at the center of an ellipse, the velocity ratio will have two maximum and two minimum values in each revolution. The teeth will act upon each other with rolling and sliding contact and the gears can be made to drive each other positively during a whole revolution.

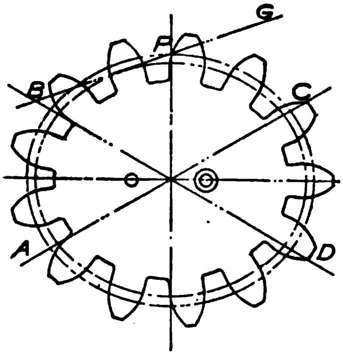


Fig. 252.

263. To Lay Out a Pair of Elliptical Gears.—Given two axes 4 in. apart which are to be connected by a pair of elliptical gears having velocity ratios that vary between the limits $\frac{3}{5}$ and $\frac{5}{3}$.

Let the gears be arranged to rotate around two fixed foci and suppose them to have standard, three-pitch, involute teeth.

Laying off the line of centers by the method of Art. 151, the gears will be found to have contact radii of 1½ in. and 2½ in. at the limits. The major axis will be 4 in., the focal distance 1 in. and the minor axis 3.88 in. long (Fig. 252).

The circular pitch will be $3.1416 \div 3 = 1.05$ in. approximately.

The circumference of the ellipse will be $3.1416 \times \sqrt{\frac{D^2+d^2}{2}} = 12.5$ in. nearly.

This will give approximately twelve teeth and if it is required that there shall be an odd number of teeth, thirteen may be taken and the circular pitch decreased the small amount necessary to give this number.

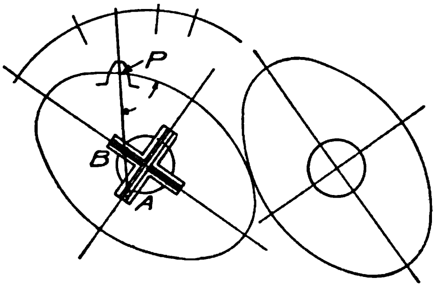


FIG. 253.

Divide the pitch ellipse into 13 equal parts, space off the teeth and draw their center lines all normal to the pitch ellipse. Calculate by Art. 197 a standard addendum and dedendum and draw the addendum and dedendum ellipses. Divide the ellipse into four sectors, *AB*, *BC*, *CD* and *DA* and find centers of circular arcs which coincide with the part of the ellipse from *A* to *B* and from *C* to *D*. Draw an involute to this base arc, make a templet for it, fasten it at the center of the arc and beginning at the major axis draw teeth between *A* and *B* and between *C* and *D*, each tooth outline being drawn symmetrical with its center line. Then find a new arc for the flat parts of the ellipse and proceed as before.

For cycloidal teeth, two describing circles will generally be sufficient, one at the major and one for the minor apex of the ellipes. The radius of such describing circles must be equal to or less than the radii of the approximate circular arcs for these positions.

If the gears are to rotate about the centers of the pitch ellipses, the center lines of the teeth may be drawn by first drawing a circle of reference outside of the pitch ellipse as shown in Fig. 253. Take the center point of any tooth on the pitch ellipse, as *P*, and on the major and minor axes locate the points *A* and *B* so that *PA* will equal the semi-major and *PB* the semi-minor axis. The center lines of the teeth are then drawn through the points *A* and *B* and the division points on the reference circle, and the tooth outlines are placed symmetrical with these lines. These teeth will not all be normal to the elliptical pitch line but if they are carefully laid out they should work fairly well.

264. Making Elliptical Gears.—With proper apparatus these teeth may be formed by any of the methods applicable to spur gears. If the rotary cutter of Fig. 183 (*a*) be used, a theoretically different size and shape will be required for each tooth on the gear. As it would be impractical to use a different cutter for each tooth, a single cutter may be selected as follows: In Fig. 252, average the radii of the circular arcs *AB* and *BC* and then select a cutter for a diameter equal to twice this average.

As a pair of elliptical gears are always made the same size the same cutter will be used for both gears.

If a gear is centered at the focus the teeth must be cut so that their center lines are all perpendicular to the pitch line, as in Fig. 251. This may conveniently be done by means of the elliptical chuck an arrangement which brings the cutter up normal to the pitch line opposite every space.

When the center of rotation is at the center of the ellipse and the latter is quite flat, the pitch lines tend to separate at four points as between *A* and *B*. This may be corrected by shortening the radius of curvature between these points so as to give the ellipse the form shown by the dotted lines.

If the number of teeth on the gear is even they must be cut so that the center line is tangent to a tooth at both ends of the major axis. If the number of teeth is odd the major axis will split a tooth at one end and a space at the other.

PROBLEMS

1. A 4-pitch, cycloidal, annular gear is 16 in. in diameter and its pinion is 5 in. in diameter. Teeth standard size and maximum angle of obliquity 15 degrees. Find the size of the describing circles to give secondary contact and design the gears.

2. One of a pair of bevel gears is 4 in. in diameter. The velocity ratio is $\frac{5}{7}$ and the angle between the axes is 90 degrees. Find the pitch cones and the diameter of the other gear.

3. If the gears in (2) are 5-pitch, find the number of teeth and the outside diameter of the gear blanks.

4. Lay out the teeth by Tredgold's approximation and show them on a side and front elevation of the gears. Involute teeth.

5. A pair of skew bevel gears are to be designed on axes that make an angle of 30 degrees, the perpendicular distance between the axes being 6 in. If the gears have 5-pitch standard, involute teeth and the velocity ratio of the driver to follower is 3 to 2, determine the gear blanks and lay out the teeth.

6. A pair of elliptical gears have a major diameter of 5 in. and a minor diameter of 3 in. If the teeth are approximately 6-pitch and the velocity ratio varies from $\frac{1}{4}$ to $\frac{4}{1}$, find the distance between the fixed foci and the number and size of teeth.

7. Design a 2-pitch pin wheel to work with a gear of 60 teeth, velocity ratio to be 5 to 1. Make the teeth and pins the same thickness at the pitch line and locate the cusp of the tooth.

8. A chain wheel is to have 14 lugs and the circular pitch is $2\frac{1}{2}$ in. If the diameter of the wire in the chain is $\frac{1}{2}$ in., find the diameter of the wheel.

9. Design two sprockets and a chain for a 1-in. pitch and a velocity ratio of driver to follower of 3 to 1. Small sprocket to be 3 in. in diameter.

10. The axes of a pair of face gears intersect at right angles, one wheel has 24 pins and the other 12. Smaller wheel to have cylindrical pins. If the circular pitch is 1 in. and the diameter of the pins is $\frac{1}{2}$ in., find the velocity ratio of driver to follower and the shape of the pins on the larger wheel.

CHAPTER XII

HELICAL GEARS

265. Classification.—The gears which are to be discussed in this chapter have been variously designated by different authors as spiral gears, helical gears, screw gears and twisted gears. These names are often confused or used synonymously.

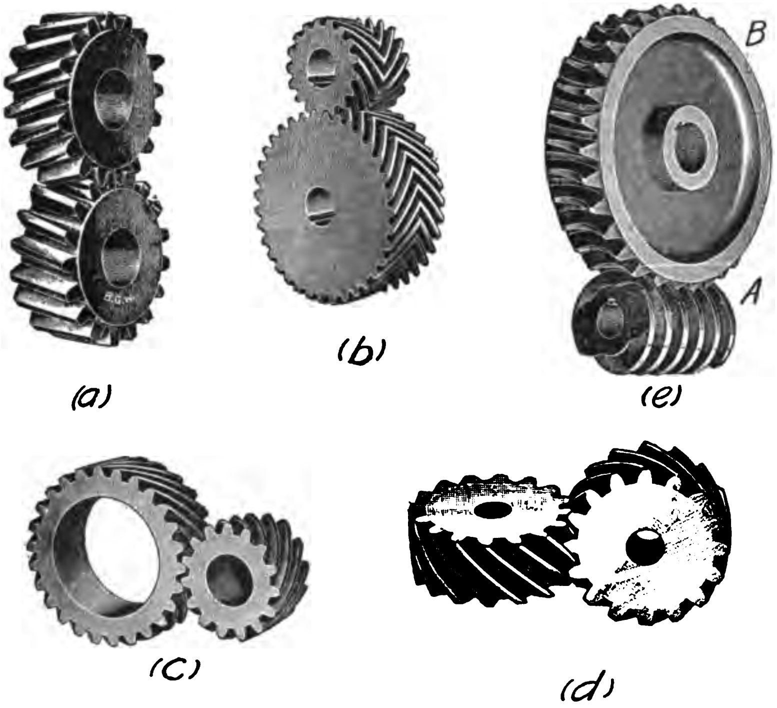


Fig. 254.

The following classification is based on the discussion by Professor MacCord.

Helical or spiral gears	Twisted gears	Single twisted gears, Fig. 254 (*a*).
		Double twisted or herring-bone gears, Fig. 254 (*b*).
	Screw gears	Worm gears, Fig. 254 (*e*).
		The type commonly called spiral gears, Figs. 254 (*c*) and (*d*).

The name *spiral gear*, which was used by Hooke to designate the twisted helical gear, is in a sense erroneous. The geometrical spiral, which is a plane curve, does not enter into its construction. *Spiral* is used rather in the loose sense of a "spiral screw thread" or a "spiral staircase." The tooth elements are in fact portions of a helix.

The slope of the helical tooth elements may be either right or left hand. A right-hand tooth, when viewed from the end of the pitch cylinder, slopes upward to the right and a left-hand tooth slopes upward to the left.

The outlines of the teeth, on the plane of reference, are of the involute form as a matter of convenience but not of necessity, any of the tooth forms that are suitable for spur gears can be used for helical gears.

Helical gears are much used when a great change in velocity is required and where noiselessness and smoothness of action at high speeds are of prime importance.

266. Relation of Axes.—The helical angle is the angle made by the tooth helix with a line parallel to the axis of the gear (α, Fig. 255).

Three arrangements of the axes are to be noted:

(1) Axes parallel.
(2) Axes askew.
(3) Axes at right angles.

Fig. 255, (*a*) and (*b*) illustrate the first arrangement. At (*a*) the helical angles are equal and one is a right-hand helix and the other left. This corresponds to the single twisted gear of Fig. 254 (*a*).

At (*b*), each gear has a right- and a left-hand helix which meet on the median planes of the wheels. This is the double twisted, or herringbone gear, shown in Fig. 254 (*b*).

In Fig. 256 (*a*) and (*b*), the axes are askew and the helical angles are unequal. At (*a*), one of the angles is right hand and the other left hand and the angle between the axes is equal to the difference of the helical angles. Fig. 254 (*c*) is a special case of the same form in which the helical angles are both 45 degrees, the axes being therefore parallel.

At (*b*) both of the helical angles are the same hand and the angle between the axes is equal to the sum of the helical angles. Fig. 254 (*d*) is a special case of this form, where the helical angles are made 45 degrees, and the axes 90 degrees apart.

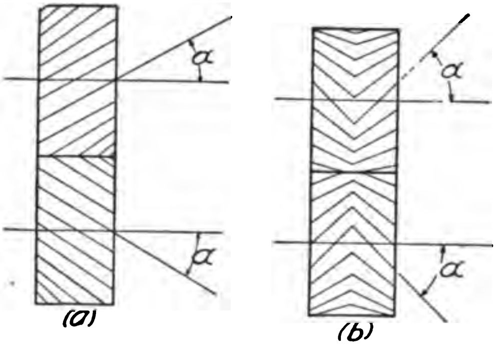


FIG. 255.

At (*c*) the helical angles are the same hand but one is much larger than the other. The sum of the angles is equal to the angle between the axes, which is always equal to 90 degrees. This corresponds to the worm gear.

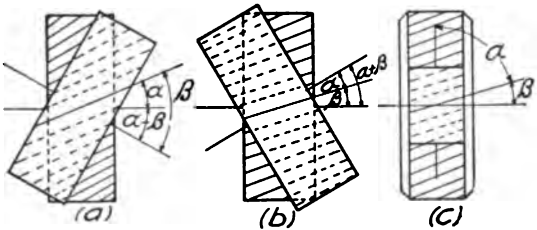


FIG. 256.

267. Twisted Gears.—If a spur gear be cut by several planes all perpendicular to the axis and each portion between the planes be then turned separately through a small angle, until the tooth elements are not continuous lines, the result will be a stepped gear, Fig. 257. When the steps are so close together that the elements are practically continuous they will be portions of a helix and the tooth will be a twisted gear tooth.

In action, these teeth do not come into contact with the mating teeth at once over their whole length, as do spur teeth. The contact begins on one side of the gear and moves across its face to the other side, along the helical elements.

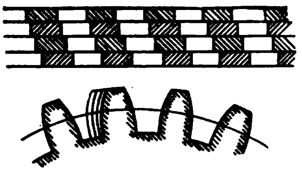


FIG. 257.

If we consider the sections of a pair of mating teeth made by a plane perpendicular to the axes, the action between these sections will be precisely like that of spur teeth. Now if the outer ends of the teeth be cut away so that these tooth sections do not make contact except at the pitch point, then the contact at that instant will be pure rolling. As contact proceeds from one side of the gear to the other, each pair of mating sections thus passes the pitch point in rolling contact, hence the whole tooth is in rolling contact.

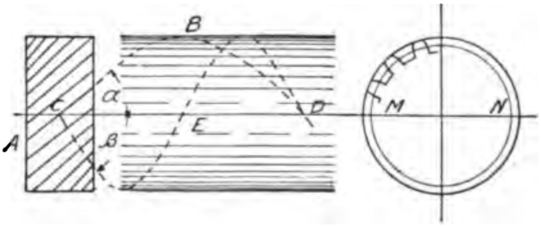


FIG. 258.

A pair of twisted teeth made as above, furnishes the single instance, referred to in Art. 46, of pure rolling contact, constant velocity ratio and positive driving action combined. These gears should not be confused with screw gears, which they resemble only in appearance. There is an exception to this, however, in the case of the screw gear of Fig. 254 (*c*), the tooth action of which is like that of a twisted gear.

268. Tooth Helix and Normal Helix.—In Fig. 258, *AB* is a tooth helix and α is the helical angle. *CD* perpendicular to *AB* is the normal helix and β is the normal helix angle. In twisted gears the helix angles are the same for both driver and follower. The plane of reference for studying the tooth outlines is here taken on the normal helix and not on a section perpendicular to the axis of the gears as with spur gears.

The smaller the helix angle is made the nearer the gear approaches to a spur gear. The best efficiency is obtained when this angle is made such size that the end of one tooth just overlaps the next one, hence the wider the face of the gear is made the smaller the helical angle required. It is good practice to make this angle 20 degrees and angles all the way from 12 to 30 degrees are common.

269. Pitch and Velocity Ratio.—The pitch of all helical gears, twisted gears included, has two forms, the true pitch and the "normal" pitch. The relation of these pitches may be briefly put in the following form:

$$\text{True pitch}\begin{cases}\text{Circular}\\ \text{Diametral}\end{cases}$$

$$\text{Normal pitch}\begin{cases}\text{Circular}\\ \text{Diametral}\end{cases}$$

The relation between the normal and the true pitch is,

$$\frac{\text{Normal pitch}}{\text{True pitch}} = \text{Cos. of the helical angle.}$$

The relation between the circular and diametral pitch is,

Circular pitch $\times$ Diametral pitch $= 3.1416$.

The true circular and diametral pitches have the same meaning that they had in spur gearing but the normal circular pitch is measured on the normal helix. (The normal diametral pitch is therefore, by definition, the number of teeth per inch of diameter on a pitch circle whose length is equal to the normal helix.) This length is measured between two intersections of the normal helix with the same tooth helix and is equal to the pitch circumference of the gear multiplied by the cosine of the helical angle.

The normal pitch is the same for both driver and follower and the contact between the teeth is made on the normal, rather than on the section perpendicular to the axis, the normal pitch is therefore used to designate the size of the teeth.

The velocity ratio of twisted gears is the same as for spur gears, that is it is proportional to the diameters or the number of teeth.

270. Herringbone Gears.—If two twisted gears, one being right hand and the other left, be fastened together side by side, the result will be the herringbone gear of Fig. 255 (*b*).

Gears of this form possess all of the advantages of single twisted gears and in addition the end thrust is eliminated by the double slope of the teeth which may make any convenient angle with the axis, up to 45 degrees.

Owing to their form, these teeth possess great strength so that it is possible to make pinions of fine pitch and consequently of small diameter. For this reason, herringbone gears make satisfactory reduction gears for high speed rotary blowers, compressors and steam turbines. Pinions with as few as 12 teeth and giving a velocity ratio of 20 to 1 are not uncommon.

For coarse pitches as in mill rolls the gears may be made by casting but when the pitch is fine and the work requires considerable accuracy some form of the cutting process is used in making them.

The gears may be cut with two rows of teeth, sloping in opposite directions, on the same blank and with considerable space left between the rows into which the cutting tool can run. They are also sometimes made by joining two single twisted gears together so that the teeth meet on a median plane or the teeth may be staggered so that a tooth on one side of the median plane is opposite a space on the other.

271. To Design a Pair of Twisted Gears.—Let it be required to design a pair of 4-pitch twisted gears for a velocity ratio of 4 to 1, axes to be 12½ in. apart.

Let α, the angle of the tooth helix be approximately 20 degrees and the pitch diameter of the gear and pinion be 20 and 5 in. respectively.

The normal circular pitch will be 3.1416 divided by $4 = 0.7854$.

The true circular pitch will be $0.7854 \div \cos \alpha = \frac{0.7854}{0.9397} = 0.8250$.

The pitch circumference of the gear will be $3.1416 \times 20 = 62.832$.

The pitch circumference of the pinion will be $3.1416 \times 5 = 15.71$.

The number of teeth on the gear, using the values given for the pitch and helical angle, will be $\frac{62.832}{0.8250} = 76.25$.

For the pinion the number of teeth will be $\frac{15.71}{0.8250}=19.05$.

Each gear must have an integral number of teeth and the pitch having been fixed it will be necessary to correct the helical angle. This may be done as follows:

$$\frac{62.832}{76}=0.8267 \text{ the corrected true circular pitch.}$$

Then the corrected $\cos \alpha=\frac{\text{normal circular pitch}}{\text{corrected true cir. pitch}}$

or

Corrected $\cos \alpha=\frac{0.7854}{0.8267}=0.9500$.

The helical angle corresponding to cos 0.9500 is $18^\circ-10'$.

The number of teeth for which to select cutter for the gear will be

$$=\frac{76^1}{\text{Cos.}^3\alpha}=\frac{76}{0.8570}=89$$

And for the pinion $=\frac{76}{0.8570}=22$.

The proportions of the gear teeth are calculated from this point on the same as for spur teeth.

272. Screw Gears.—Screw gears resemble twisted gears in having helical tooth elements but the similarity ends there for the contact and velocity ratio are quite different.

If a pair of spur or twisted gears be cut by a plane perpendicular to the axes, a pair of mating tooth sections in that plane, will at the instant of passing the pitch point, be moving in the same direction with respect to that point. A similar pair of mating sections on two gears will be moving in different directions at the instant of passing the pitch point and these directions will depend upon the helical angles. It follows therefore that two screw gear teeth cannot pass the line of centers, in contact, without some sliding upon each other along the contact elements. This sliding

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[1] The cutter is selected, not for the actual number of teeth on the gear, but for the number of teeth that a spur gear of the same pitch would have if its radius were equal to the radius at the flattest part of an ellipse. The ellipse being formed by taking a section of the gear on a plane tangent to the normal helix. A demonstration of this may be found in *Machinery* for April, 1908.

is illustrated by Fig. 259 where *AB* represents a pair of helical gear teeth, located between the axes and in contact at the point *P*.

If gear *C* is driving in the direction shown, the normal pressure between the teeth tends to move the tooth of *D* in the direction *PN*, but the tooth is constrained by the axis of its pitch surface, to move in the direction *PK*. At the same time the driving tooth on *C* is constrained to move in the direction *PL*. Then *NK* represents the sliding of the point of contact on *D* and *NL* represents the sliding on *C* and *NL*+*NK* is the total amount of sliding between the teeth.

As the angle between the axes, $\theta+\theta'$, which is equal to the angle *LPK*, increases, the amount of slipping between the teeth increases, as is shown by the lengthening of *LNK*.

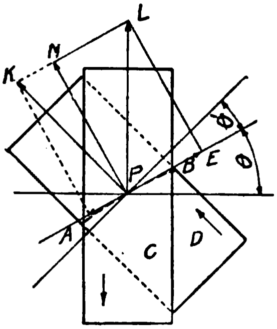

Fig. 259.

273. Worm Gears.—A worm is a helical gear of relatively small diameter having a helical angle so large that a tooth makes a complete convolution about the axis in a distance which is equal to the circular pitch of the teeth. The mating gear is called a worm wheel, its diameter is large compared with that of the worm and each tooth is a short portion of a helix the angle of which is the complement of that of the worm. The whole mechanism is popularly known as the "endless screw."

The worm is generally the driving element and unless the helical angle of the wheel is greater than the angle of repose of the materials in contact, the wheel cannot be made to drive the worm. This property is in certain cases one of the advantages of the mechanism, for example in the windlass or elevator, it sometimes serves as a locking device and prevents the machine from running backward.

The principal defect of the worm gear is its wastefulness of power when used for transmission, especially for slow and powerful motions. For light loads and in adjustment motions where the force transmitted is of no consequence it is less objectionable and makes a very useful device.

There are three classes of worm gears: (I) Wheel with straight teeth. The teeth set on the wheel at an angle which is the complement of the helical angle of the worm. (II) Wheel with helical teeth fitted accurately to the thread of the worm. (III) The Hindley or hour glass worm gear.

274. Contact of Worm Gears.—Fig. 260 shows a section of a worm and wheel in mesh, (*a*) being a section at *LL* perpendicular to the axis of the worm and (*b*) a section at *AA* through the median plane of the wheel and containing the axis of the worm. Supposing the gears to have involute teeth, then this section of the worm will be like that of an involute rack and the corresponding section of the worm wheel will be like that of an involute gear proper to mesh with the rack. Pass a number of planes *D*, *B*, *C*, and *E*, parallel to *AA* and very close together. Then the tooth sections cut from worm and wheel by any two consecutive planes will be practically alike but each pair of mating sections, as shown at (*b*), will be a little in advance of the preceding pair. This means that if the worm be rotated on its axis, the driving effect on the wheel will be precisely like that of a rack driving a pinion, contact between the tooth sections taking place along the line *FG* and moving from root to point of tooth. Contact between the tooth sections of worm gears, however, does not take place on planes parallel to, or through the axis of the worm, but on planes which are normal to the helical tooth elements. Since the angle between the axial and normal planes is small the distinction is of no great practical importance.

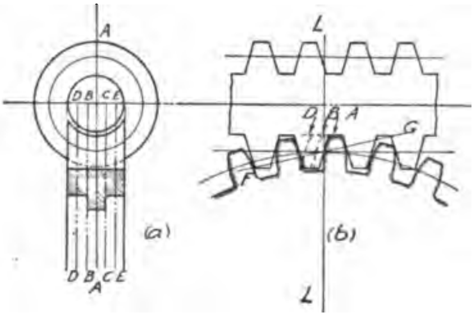

FIG. 260.

We have already stated that the line of action for tooth sections on *AA* is the line *FG*.[1] On sections *BB* or *CC* this line of action will be curved as shown at *HH*, Fig. 261, and the farther the section is taken from the median plane the more the form the line of action will depart from that of FG.

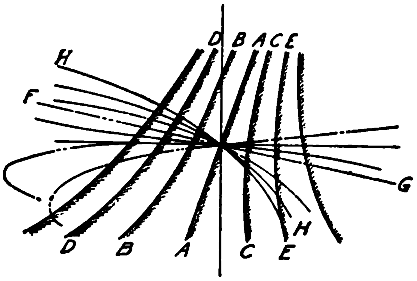

Fig. 261.

If all these lines of action be projected horizontally upon the worm the result will be an irregular figure *KA'K'A''*, Fig. 262 (*a*). If projected vertically upon the worm, the surface shown by Fig. 262 (*b*) would result. This might be called the surface of contact, that is it is the surface that would be generated by the

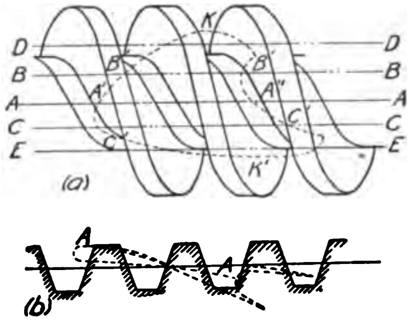

Fig. 262.

line of contact between the worm thread and one wheel tooth as the latter moves through the arc of action. The contact at any instant will be on a line formed by the intersection of this surface and the curved faces of the teeth. At the instant of passing the line of centers this contact line will be similar to *KK'*, Fig. 263 (*a*).

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[1] This discussion was first published in the American Machinist.

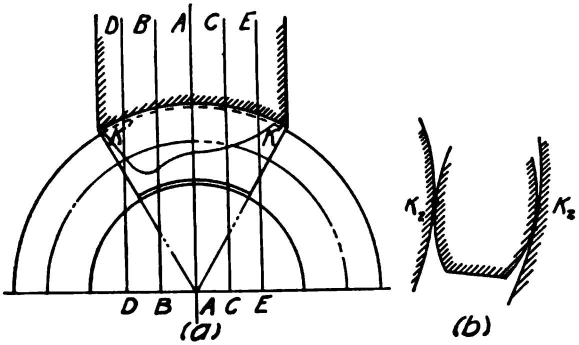

FIG. 263.

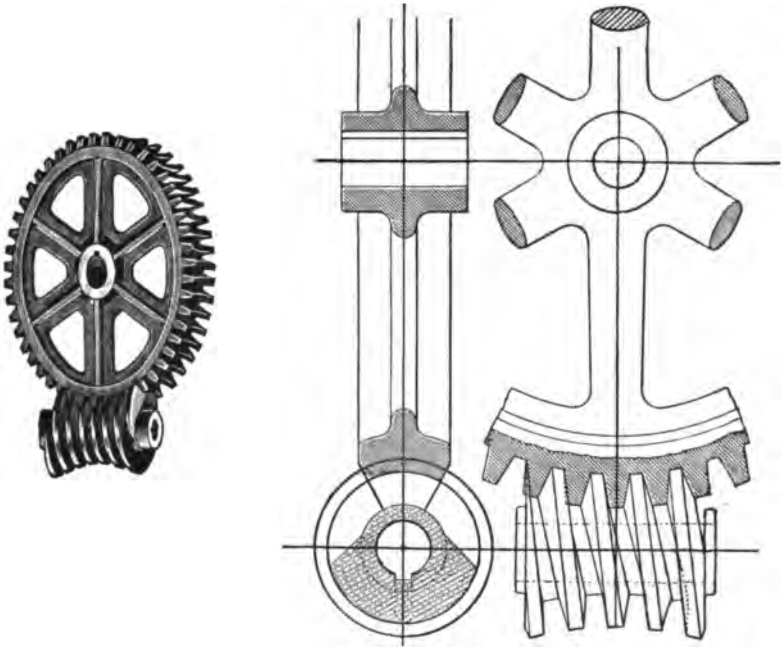

FIG. 264.

Owing, however, to the elasticity of the materials in contact this line will have some breadth so that the contact is actually on a surface of no great width, as at K_2K_2 in Fig. 263 (*b*).

In the Hindley worm gear, Fig. 264, the pitch surface of the worm instead of being a cylinder is made so as to fit the circumference of the wheel. In the section shown, the addendum, pitch and dedendum lines of the worm are circular arcs drawn with radii equal to those of the wheel, and the sections of the threads are placed radially to the wheel. It is claimed that this arrangement allows the sections of the worm thread to completely fill the spaces between the wheel teeth so as to give surface contact. In consequence of this the Hindley worm gear is said to have greater smoothness of action, greater durability and efficiency.

While the contact may be somewhat nearer to surface contact when the gear is new, very accurately made and adjusted, in ordinary cases it is probably not very different from the worm gears of Class II. After the gear becomes worn or when not adjusted so that the smallest diameter of the worm is over the center of the wheel the worm threads are likely to interfere badly with the teeth of the wheel.

275. Pitch, Velocity Ratio and Directional Relation, Worm Gears.—Pitch, as applied to worm gears should be taken to mean the circular pitch. On the worm it is called the axial divided pitch. In Fig. 265, M is the circular pitch of the worm wheel and R is the axial divided pitch of the worm. These distances are equal and correspond to the circular pitch of a spur rack and pinion.

When the worm makes one complete revolution it drives a wheel tooth through the distance M. Therefore when the worm has only one thread, or tooth, the velocity ratio of the worm to the wheel is the reciprocal of the number of teeth on the wheel. Thus, if a wheel has 40 teeth the velocity ratio will be be $\frac{1}{40}$.

Sometimes the worm is made with two or more threads in a single pitch distance as if a single thread were divided into several parts. The above velocity ratio would then be $\frac{2}{40}$ or $\frac{3}{40}$ as the case might be.

The direction of rotation of a worm wheel is at right angles to that of the worm and in Fig. 265, for example, if the worm has right-hand threads and be rotated when viewed from the right-hand end, in the clockwise direction, the wheel will also turn in the clockwise direction. If the thread of the worm be left hand or if it be placed below the wheel the directional relation will be reversed.

276. Design of a Worm and Wheel.—Suppose it is required to lay out a worm and wheel with a velocity ratio of 40 to 1. Let the worm have a single thread and the circular or axial pitch be 2 in.

The worm wheel will have 40 teeth and its diameter will be

$$\frac{40\times2}{3.1416}=25.5.$$

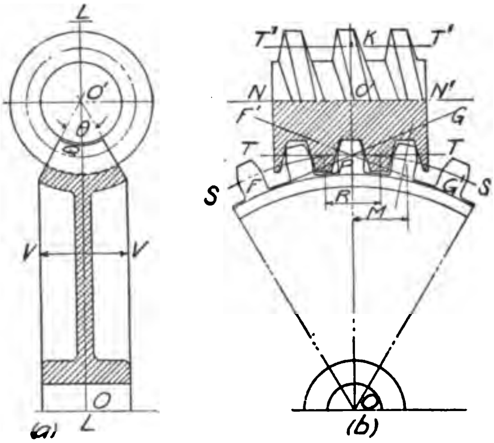

FIG. 265.

The diameter of the worm at the pitch line may be taken at from one and one-half to three times the pitch, let us say 6 in. in this case.

Lay off the line of centers OO', Fig. 265, equal to $\frac{25.5}{2} = 15.62$ in. Through O' draw NN' the center line of the worm, then draw the pitch lines TT and $T'T'$, 6 in. apart. Draw SS the pitch circle of the wheel, 25.5 in. in diameter. Calculate the addendum and dedendum as for spur gears and draw in the tops and bottoms of the teeth as shown.

Take a section LL through the median plane of the worm and wheel. This section will be like that of an involute spur wheel and rack.

Draw the pressure lines *FG* and *F'G'* making an angle of say, 15 degrees with the tangent to the pitch circle and lay off the base circle of the wheel tangent to *FG* or *F'G'*. Describe an involute on this base circle, make a template to fit the involute curve, space off the teeth on the pitch line and draw in the wheel tooth outlines. The sides of the worm teeth on this section will be straight lines perpendicular to the pressure lines. For any other section the tooth outlines may be found by projection, if necessary. The face *VV* of the wheel may be made from one and one-half to two times the circular pitch. The angle θ is commonly made 60 degrees. *OQ* is the throat diameter of the wheel.

277. Spiral Gears and their Contact.—The term, "spiral gear" is here used to designate the class of helical gears shown in Fig. 256 (*a*) and (*b*).

The helical angles of these gears are made anywhere from 12 to 45 degrees, the larger angle tending to give the best efficiency with the least wear.

Contact between the teeth of spiral gears is almost undefinable, it appears to be at once the pure sliding contact of the worm gear and the mixed rolling and sliding contact of the spur gear.

Assume the driver to be cut by a plane normal to the teeth and the follower to be cut by several such planes, all very close together. A section of the driver's tooth on the cutting plane will make contact at a particular instant, with a section in plane number one, through the follower's tooth, the contact tending to progress along a line of action similar to *FG* in Fig. 265. At the next instant this section of the driver's tooth will be in contact with the section in plane number two of the follower's tooth and so on to each succeeding section until that section of the driver's tooth has passed out of the arc of action.

The fact of the contact being on a normal instead of an axial section is made more prominent with this class of gears than with worm gears because the angle between the normal and axial sections is greater.

278. Pitch and Velocity Ratio of Spiral Gears.—The true circular pitch of a spiral gear is measured on the pitch circumference of the gear and in a direction which is perpendicular to the axis. The true diametral pitch is equal to 3.1416 divided by the true circular pitch.

The normal circular pitch is measured on the normal helix and is equal to the length of the helix divided by the number of teeth. The length of the normal helix is equal to the pitch circumference of the gear multiplied by the cosine of the helical angle θ, Fig. 266. Then the normal diametral pitch will be the number of teeth per inch of a diameter on a pitch circle, equal in length, to this normal helix. Its value may also be found by dividing 3.1416 by the normal circular pitch.

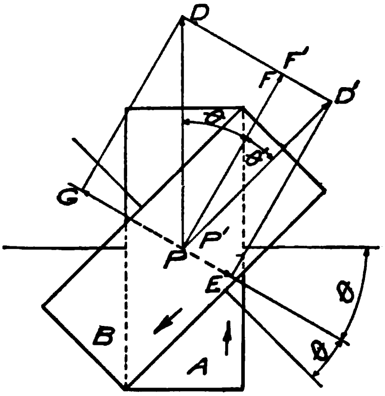

FIG. 266.

The normal circular pitch governs the size of the teeth and is the same for both gears of a pair but the true circular pitch is not necessarily the same on driver and follower.

The number of teeth on the gear is equal to the pitch circumference divided by the true circular pitch.

In designing a pair of spiral gears, that one which has the larger helical angle should be made the driver.

Let *A* and *B*, Fig. 266, have helical angles of θ and θ', while *R* and *R'* represent their radii and *N* and *N'* the revolutions per unit of time. Suppose the linear velocity of a point of contact *P*, on the driver *A*, to be represented by the length *PD*. Resolving this motion into its components, *PG* equals the velocity of sliding along the tooth elements and *PF* along the normal helix, represents the motion transmitted. PF is the same for both driver and follower.

Now $$PD=2\pi RN,$$

and from the figure,

$$PF=PD\cos\theta=2\pi RN\cos\theta.$$

Also, since $PF=P'F'$,

$$P'F'=2\pi RN\cos\theta.$$

From the figure, the linear velocity of the point of contact on B is shown by the distance $P'D'$.

Again $P'F'=P'D'\cos\theta'$ or by transposing, and substituting for $P'F'$ its value,

$$P'D'=\frac{2R\ N\cos\theta}{\cos\theta'},$$

The number of revolutions made by B is equal to the linear velocity of P' divided by the circumference.

That is,

$$N'=\frac{2\pi RN\cos\theta}{\cos\theta'}\div 2\pi R'=\frac{N(R\cos\theta)}{R'\cos\theta'}$$

and

$$\frac{N}{N'}=\frac{R'\cos\theta'}{R\cos\theta} \qquad \text{(a)}$$

Expressed in words, the angular velocity ratio of a pair of spiral gears is inversely proportional to the product of their radii by the cosine of their helical angles.

If both θ and θ' are made equal to 45 degrees this formula becomes $\frac{N}{N'}=\frac{R'}{R}$ which is the same as for spur gears.

279. Design of Spiral Gears.—This problem may be stated in either of two ways as follows:

(a) The diametral pitch, the number of teeth and the angle made by the axes given to determine the helical angles, the diameter of the gears and the distance between the axes.

(b) The distance between the axes, angle made by the axes, the velocity ratio and the pitch given to find the diameters of the gears, the number of teeth on each gear and the helical tooth angles. The limitation being also given that the teeth shall be cut with standard stock cutters.

Case 1.—Let it be required to design a pair of gears having 18 and 45 teeth, the angle made by the axes being 60 degrees and the normal pitch 6.

Suppose the larger gear to be the driver and to have a helical angle of 40 degrees. Then if the gears are both made the same hand, as in Fig. 256 (*b*), the helical angle of the follower will be $60-40=20$ degrees.

The true diametral pitch of the driver will be

$$\frac{6}{\cos 40°}=\frac{6}{0.766}=7.83.$$

Its pitch diameter will be

$$\frac{45}{7.83}=5.75 \text{ in.}$$

The true diametral pitch of the follower will be

$$\frac{6}{\cos 20°}=\frac{6}{0.939}=6.4.$$

Its pitch diameter will be

$$\frac{18}{6.4}=2.81 \text{ in.}$$

The distance between the axes will be

$$\frac{5.75+2.81}{2}=4.28 \text{ in.}$$

Case 2.—Suppose in this case that the gears are to have an angular velocity ratio of $\frac{\text{Driver}}{\text{Follower}}=\frac{5}{7}$. Suppose the angle between the axes to be 10 degrees and the perpendicular distance between them to be 6 in.

If one of the gears be made with a right-hand helix and the other left, as in Fig. 256 (*a*), the helical angles may be 40 for the driver and 30 for the follower.

By formula (a), Art. 164, $\frac{N}{N'}=\frac{R' \cos \theta'}{R \cos \theta}$.

But $\frac{N}{N'}=\frac{5}{7}$ in the present example, therefore

$$\frac{R'}{R}\frac{\cos\theta'}{\cos\theta}=\frac{5}{7}, \text{ from which}$$

$$\frac{R'}{R}=\frac{5\cos 30}{7\cos 40}=\frac{5\times 0.866}{7\times 0.766}=0.80+.$$

We have then to determine two radii whose sum is 6 and whose ratio is 0.81. These radii will be 3.32 for the driver and 2.68 for the follower and the corresponding diameters will be 6.64 and 5.36 in.

The cutter is to be selected from the normal diametral pitch, any one being taken that is coarse enough to give the required strength to the teeth. Suppose an 8-pitch cutter to answer the present requirement.

Then the true diametral pitch of the driver will be

$$\frac{8}{\cos 40^\circ}=\frac{8}{0.766}=10.4,$$

and the true diametral pitch of the follower will be

$$\frac{8}{\cos 30^\circ}=\frac{8}{0.866}=9.25.$$

The number of teeth on the driver will be $6.64\times 10.4=69$ and the number of teeth on the follower will be $5.36\times 9.25=50$.

In case the number of teeth does not come out a whole number a fraction of a tooth may be added or subtracted and a slight change made in either the helical angle or the velocity ratio.

The number of teeth for which to select the cutter for the driver will be

$$\frac{69}{\cos^3 40^\circ}=\frac{69}{0.450}=153$$

and for the follower

$$\frac{50}{\cos^3 30^\circ}=\frac{50}{0.650}=77$$

280. Manufacture of Helical Gears.—The various kinds of helical gears are made by one of three methods: (1) casting, (2) cutting, or (3) hobbing.

Casting the gears is a somewhat more difficult process than for spur gears because of the difficulty of getting the teeth of the pattern out of the sand. Herringbone gears for mill rolls and heavy worm gears for marine windlasses are sometimes made by this process.

The teeth of helical gears may be made by either the milling or planing process. The operation is not different from that of cutting spur teeth except that the gear blank must be given a slow rotary motion while the tool is cutting so as to give the helical form to the teeth.

The following difficulties and inaccuracies are met with in cutting helical gear teeth:

(a) When the gears are of small diameter, there is a difference between the helical angle at the top and bottom of the teeth. It is common to take this helix at the pitch circle, but as this gives a slightly undercut tooth some makers prefer to use the helix at the bottom of the tooth.

(b) The rotation of the blank must be such as to give exactly the right angle to the tooth helix, otherwise the contact between a pair of teeth will be bad.

(c) With milled teeth it is generally convenient to use the same standard cutter as for spur gears of the same pitch. Since the number of teeth in a gear is not always in proportion to the diameter, the circumference cannot usually be divided into a whole number of teeth by the standard cutters at hand, the diameters or spiral angles must therefore be readjusted so as to give an integral number of teeth.

Hobbed gears are made as already described under spur gears except that the hob is set at a different angle to the plane of rotation of the wheel to allow for the angularity of the teeth. In hobbing worm gears, the hob and wheel are so mounted that they can be rotated separately and their axes at the same time gradually brought nearer together. The hob and wheel should be driven independently of each other and it is good practice to first gash the teeth with a circular cutter before using the hob. The hob should be kept sharp. If these points are not attended to, unequally spaced teeth may result or the number of teeth produced may be different than what was intended. The worm wheel fails to follow up properly when the worm is first starting the cut if the hob is allowed to drive the wheel blank. When hobbed teeth are rough, showing "flats" or ridges along their elements, they may be much improved by grinding. This can be done by running the wheel with a hardened worm using graphite or some similar substance between the teeth.

In cutting the worm, the side of the cutting tool for involute teeth should be straight and the tool should be set to make an angle with the axis, equal to the helical angle of the worm.

PROBLEMS

1. Design a pair of herringbone gears for a velocity ratio of 10 to 1, the smaller gear being the driver and having 24 4-pitch, involute teeth. Helical angle to be 25 degrees on the driver. Allow ½ in. on the median plane for the cutter to run out, and determine the best width of face and the proper cutter for the teeth.

2. The radii of a pair of spiral gears are 2 and 4 inches respectively, the spiral angles are, driver, 31 degrees, right, follower, 17 degrees left. If the pitch is 8, find the number of teeth on each gear and the velocity ratio.

3. Find the velocity ratio in (2) if the axes are parallel. If they make an angle of 90 degrees.

4. A pair of 6-pitch spiral gears are 4 and 7 in. in diameter and transmit a velocity ratio of 3/2. Find the number and length of teeth, the spiral angle, the normal and real circular pitch and the angle between the shafts, supposing the spiral angles to be both right hand.

5. A 4-pitch worm gear has 60 teeth, its worm is single pitch. Find the velocity ratio, diameter of gear and size of teeth.

6. Do worm gears have point, line or surface contact? Prove.

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